



Universidade de São Paulo
Instituto de Astronomia, Geofísica e Ciências Atmosféricas
Departamento de Astronomia

Exam of Proficiency in Mathematics and Physics

Required for admission in the Graduate Course of Astronomy

May 3rd, 2016

Name:

Instructions to the candidate (read carefully):

- The exam is individual, without any consulting. You may use a calculator but you **may not** use a cellphone or access the Internet. The total exam duration is up to 4 hours.
- The exam should be done using a **pen** (do not use a pencil). Please, write down your name on **each** page and number them.
- If you are doing the exam outside IAG/USP, use only **one side** of each white sheet of paper. Remember to always put the number of the question you are answering. Leave at least a 2 cm (0.8 inch) margin at all sides. Remember that paper sheets with your answers will be scanned to be sent to the committee.

1. (Quantum mechanics)

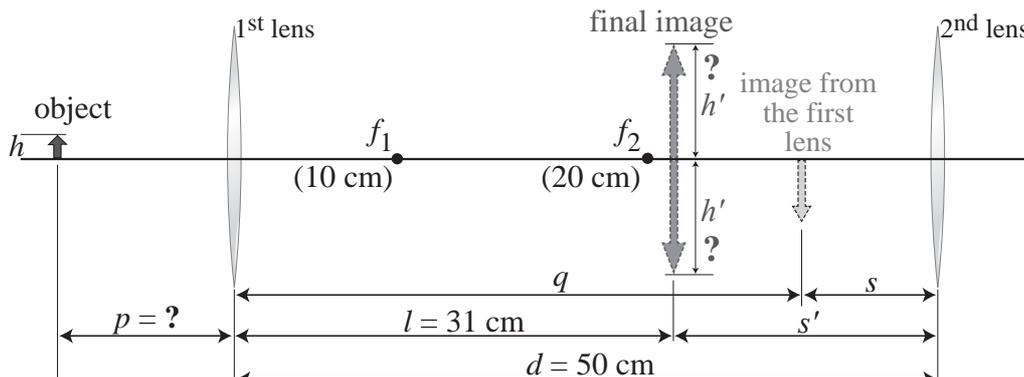
A given nucleon of rest energy of 940 MeV is bound in an atomic nucleus of radius 1 fm ($= 10^{-15}$ m). Use Heisenberg's Uncertainty Principle to estimate the kinetic energy (in MeV) of that nucleon.

2. (Relativity)

In the inertial system S , an event is observed to take place at point A on the x -axis and 10^{-6} s later another event takes place at point B, on the same axis, 900 m further down. Find the velocity of S with respect to S' in which these two events appear simultaneous.

3. (Optics)

Two converging lenses with focal lengths $f_1 = 10$ cm and $f_2 = 20$ cm are positioned 50 cm apart, as shown in the figure below. The final image is to be located between the lenses, at the position indicated, 31 cm to the right of the first lens. **(A)** How far to the left of the first lens should the object be? **(B)** What is the overall magnification, h'/h ? **(C)** Is the final image upright or inverted?



4. (Statistics)

The human blood has 4 groups: A, B, AB, and O. Independently of each group, an individual may have the so-called Rh factor, either positive or negative (Rh+ or Rh-).

For a given population we have the following group distribution:

A	B	AB	O
45%	9%	3%	43%

For each group, the population distribution with respect to the Rh-factor is the following:

Group	A	B	AB	O
Rh+	87%	78%	67%	86%
Rh-	13%	22%	33%	14%

(A) What is the probability that an individual randomly chosen be an universal donator? (blood type O-).

(B) What is the probability that an individual randomly chosen have Rh-.

5. (Waves)

You want to evaluate a new wire for a piano. This wire has a length of 3.00 m and a specific linear density $\mu = 0.00250 \text{ kg m}^{-1}$. Testing the wire, you found two adjacent resonant frequencies at 252 Hz and 336 Hz. **(A)** What is the fundamental frequency of the wire? **(B)** What is the traction F_T in Newtons applied to this wire? Hint: The propagation velocity of a transversal wave in a wire is $\sqrt{F_T/\mu}$.

6. (Mechanics)

A particle of mass m is subjected to two forces: a central force f_1 and a frictional force f_2 , with

$$\vec{f}_1 = \frac{\vec{r}}{r} f(r) \quad \text{e} \quad \vec{f}_2 = -\lambda \vec{v}; \quad (\lambda > 0),$$

where \vec{v} is the velocity of the particle and λ is a constant. If the particle initially has angular momentum J_0 about $r = 0$, find its angular momentum for all subsequent times.

Hint: The use of polar coordinates simplifies the solution.

7. (Logic)

Euler Problem: a farmer has bought horses and oxen and paid a total amount of \$1770. If a horse costs \$31 and an ox costs \$21, how many horses and how many oxen has the farmer bought? Note that the amount of horses and oxen are positive integers.

8. (Calculus)

Prove that $(\cos \theta)^p \leq \cos(p\theta)$ for $0 \leq \theta \leq \pi/2$ and $0 < p < 1$.

9. (Thermodynamics)

Define the mean free path of a gas both mathematically and in words. Calculate the mean free path of a molecule in a gas if the number of collisions per second is 2×10^{10} and the mean molecular velocity is 1000 m s^{-1} .

10. (Electromagnetism)

In the Bohr model of hydrogen atom the electron revolves in a circular orbit of radius 0.53 \AA with a time period P of $1.5 \times 10^{-16} \text{ s}$. Find the current corresponding to this movement.

Additional data: $c = 2,998 \times 10^5 \text{ km s}^{-1}$; $h = 6,626 \times 10^{-34} \text{ m}^2 \text{ kg s}^{-1}$,

$\hbar = h/2\pi = 6,58211889 \times 10^{-22} \text{ MeV s}$.

Electron charge = $1,6 \times 10^{-19} \text{ C}$

Lorentz transformations:

$$\gamma = 1/\sqrt{1 - (v/c)^2}$$

$$x' = \gamma(x - vt) \iff x = \gamma(x' + vt')$$

$$t' = \gamma(t - vx/c^2) \iff t = \gamma(t' + vx'/c^2).$$

Lens equation:

$$\frac{1}{\text{distance to objet}} + \frac{1}{\text{distance to image}} = \frac{1}{\text{focal distance}}$$

Velocity and acceleration in polar coordinates (r, θ) :

$$v_r = \dot{r} \quad ; \quad a_r = \ddot{r} - r\dot{\theta}^2$$

$$v_\theta = r\dot{\theta} \quad ; \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
