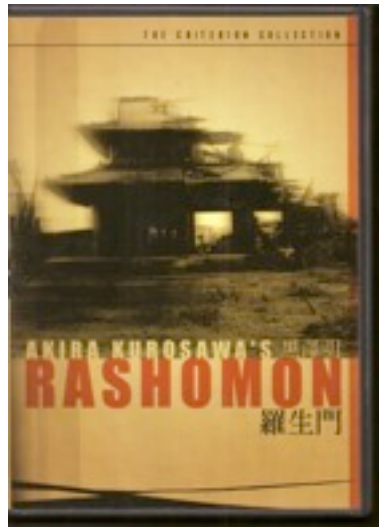


# **Crash Course in Probabilistic Seismic Hazard Analysis (PSHA)**

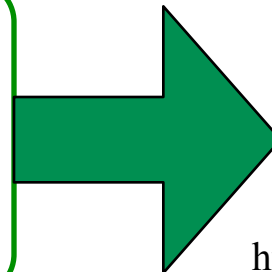
## **Part 1**

# Content



## mathSHA

seismic hazard analysis using *Mathematica*



<http://www.wolfram.com/products/player/>

# Seismic hazard is shaking hazard



**Don't think about earthquakes first,  
think about ground motion first!\***



**\*)But don't forget the earthquakes**

# Key aspects of basic PSHA



PSHA provides models (PSHMs) for shaking hazard in terms of chosen ground motion intensity parameters

PSHMs are probabilistic in nature: ground motion is treated as a **random variable**

Questions that can be addressed with PSHMs are e.g.

- What is the ground motion which is expected to be exceeded with a particular probability at a particular site within a particular time interval?
- What is the probability for a particular ground motion value to be exceeded at a given site of interest within a particular time interval?

# PSHA in a nutshell



Modern PSHA deals with probabilistic models for seismically generated ground motion. It combines concepts and methods from seismology, earthquake engineering, and probability theory.

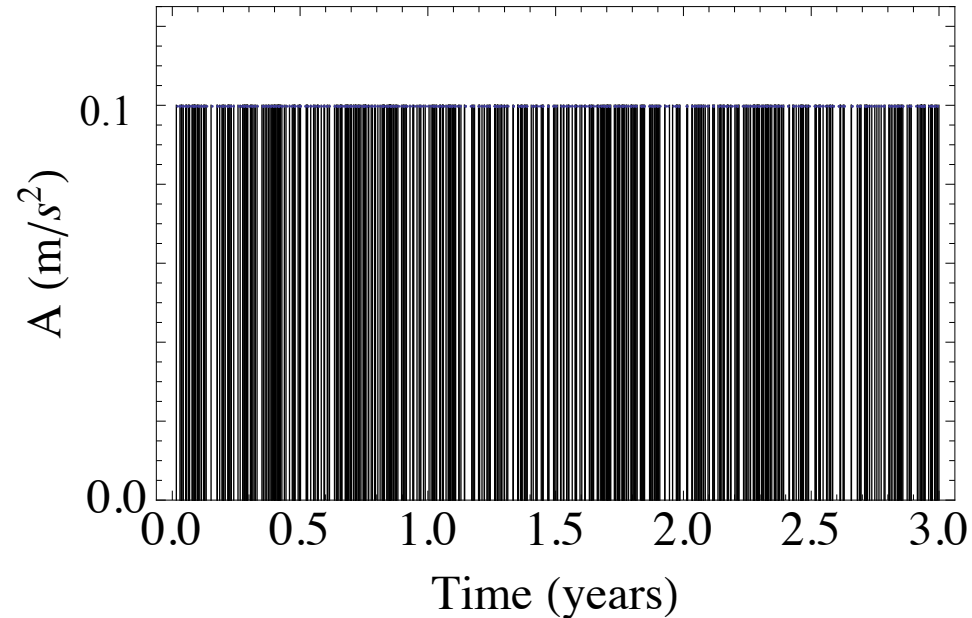
For critical facilities, modern PSHA includes also a systematic assessment of (epistemic) uncertainties.

Focus here: **Conceptual understanding**

*Don't be afraid to do simple things*  
- R. Adams

# Single idealized source

$$a_i = 0.1 \text{ m/s}^2 \quad (\text{for all } i)$$



$$\lambda = \text{average \# events / year} = 200$$

**Exercise 1: What is the expected daily rate at which certain ground motion levels are reached or exceeded?**

# Exceedance rate & exceedance probability

**Rate:** How often in absolute sense (per chosen time interval)

**Probability:** How often in relative sense (e. g. in comparizon to a large number of similar cases)

## Examples:

How many heads in 500 trials?



$$500 \cdot 0.5 = 250$$

How many 4s in 600 trials?



$$600 \cdot \frac{1}{6} = 100$$

Expected number: # trials · prob(single event)



# Single idealized source

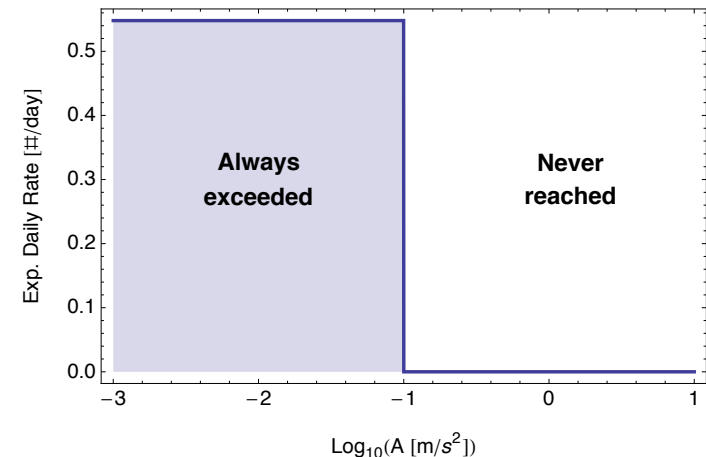
$$\lambda = \text{average \# events / year} = 200 \quad a_i = 0.1 \text{ m/s}^2 \quad (\text{for all } i)$$

***What is the expected daily rate at which certain ground motion levels are reached or exceeded?***

Expected daily rate of occurrence of blasts is:  $\# \text{ blasts/day} = 200/365 = 0.55$

Each of them generates  $0.1 \text{ m/s}^2$

$$\begin{aligned} P(a \text{ is reached or exceeded}) &= 1 \quad (\text{for all } a \leq 0.1 \text{ m/s}^2) \\ &= 0 \quad (\text{for all } a > 0.1 \text{ m/s}^2) \end{aligned}$$

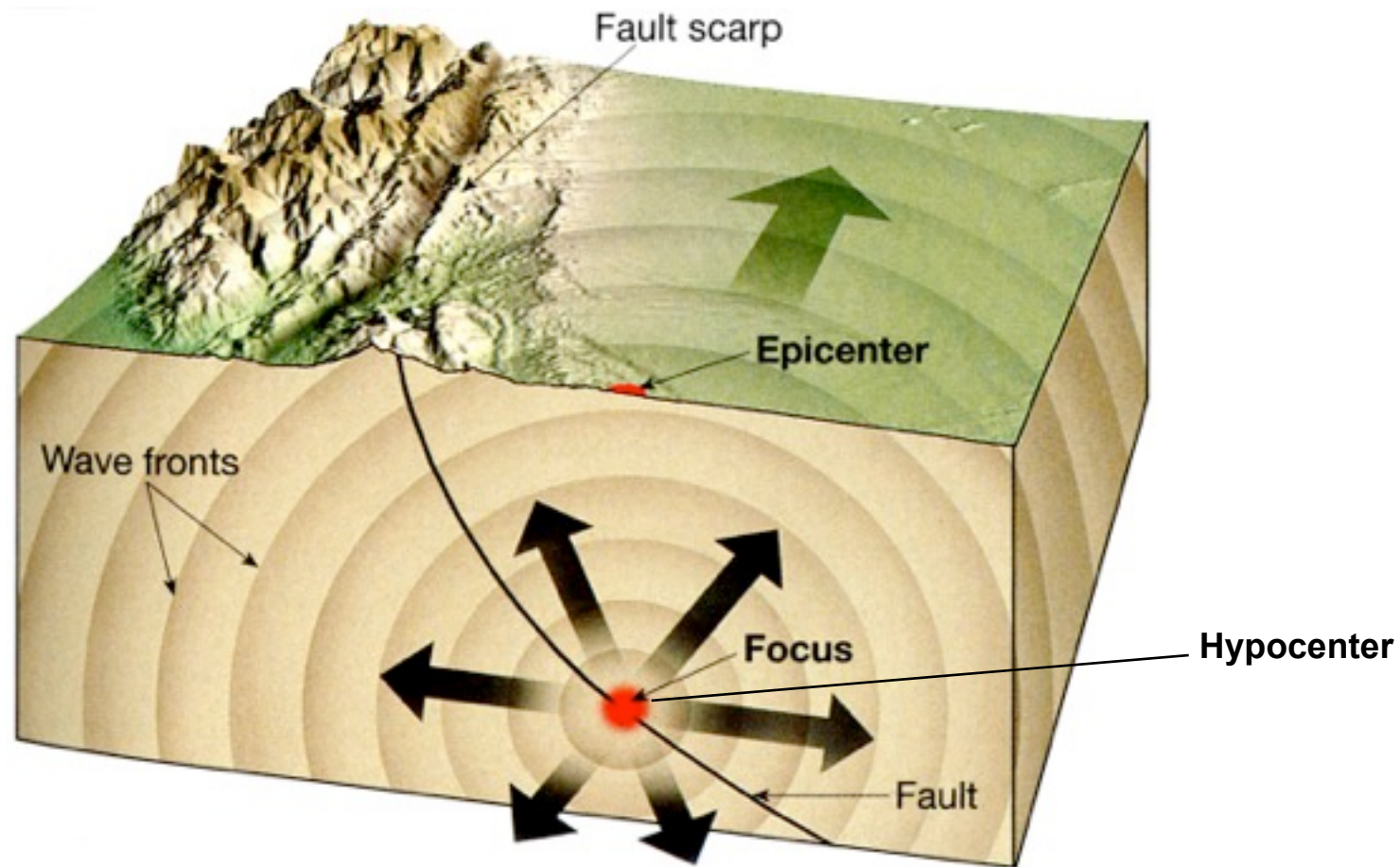


$$\begin{aligned} \text{ExpDailyRate} &= 0.55 \cdot 1 = 0.55 \quad (\text{for all } a \leq 0.1 \text{ m/s}^2) \\ &= 0.55 \cdot 0 = 0 \quad (\text{for all } a > 0.1 \text{ m/s}^2) \end{aligned}$$

# Single earthquake source



# What happens during an earthquake?



# From myths to models

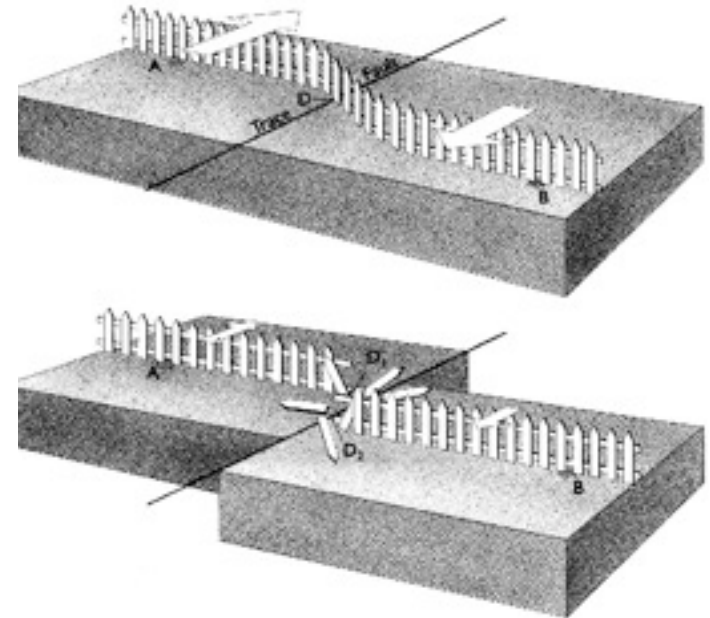


# San Francisco earthquake, 18. of April 1906

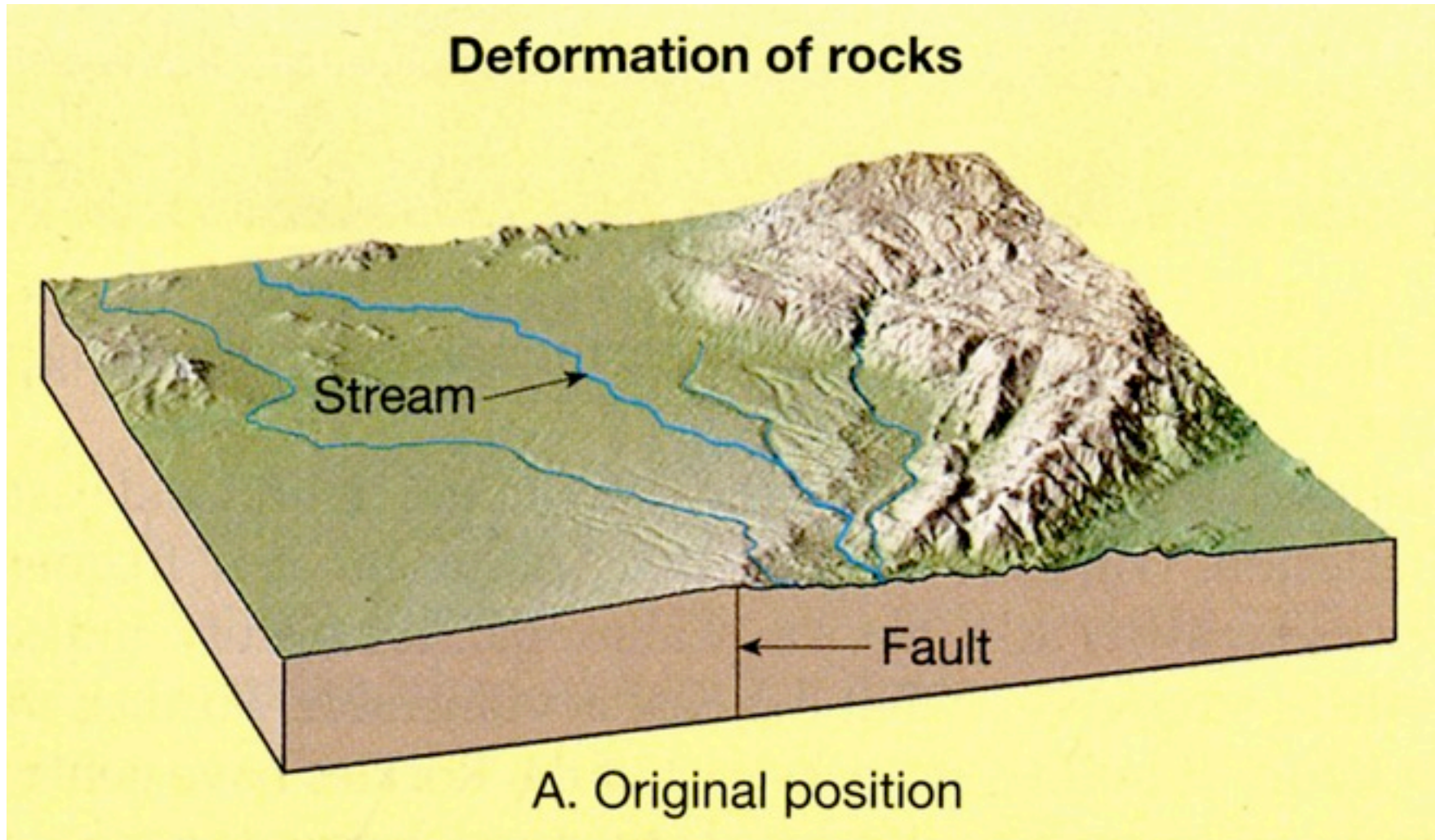




# From a shifted fence to the rebound hypothesis

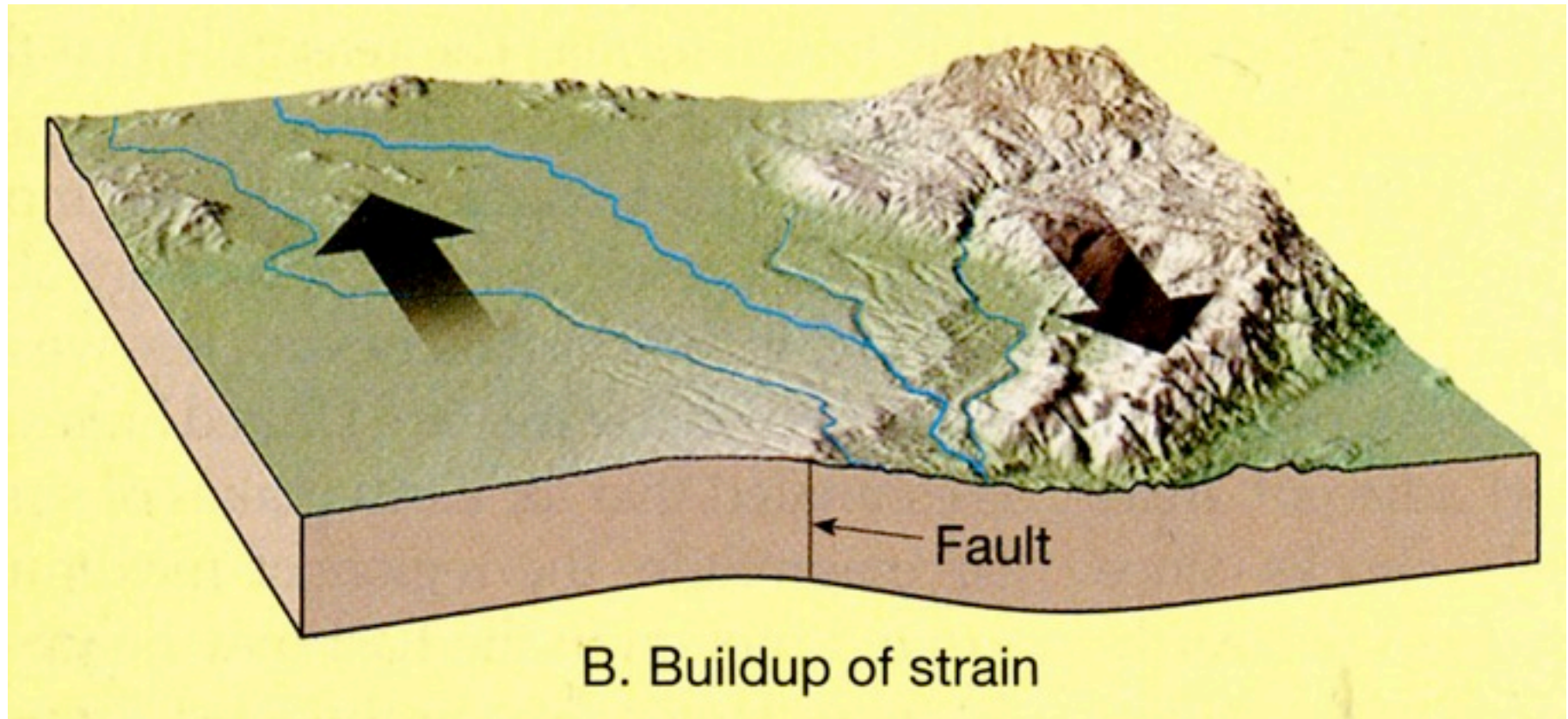


# Principle of the rebound hypothesis



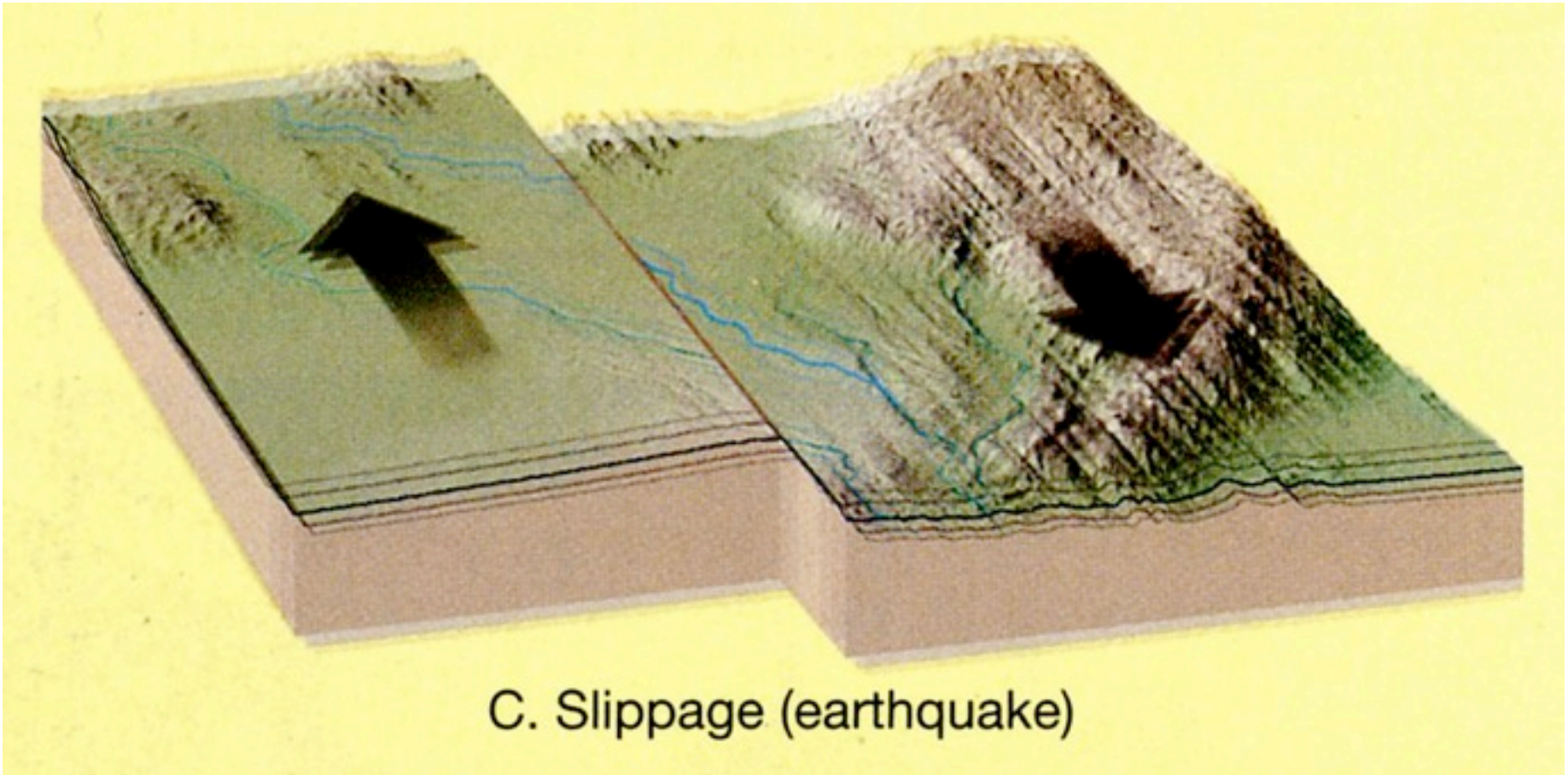


# Principle of the rebound hypothesis

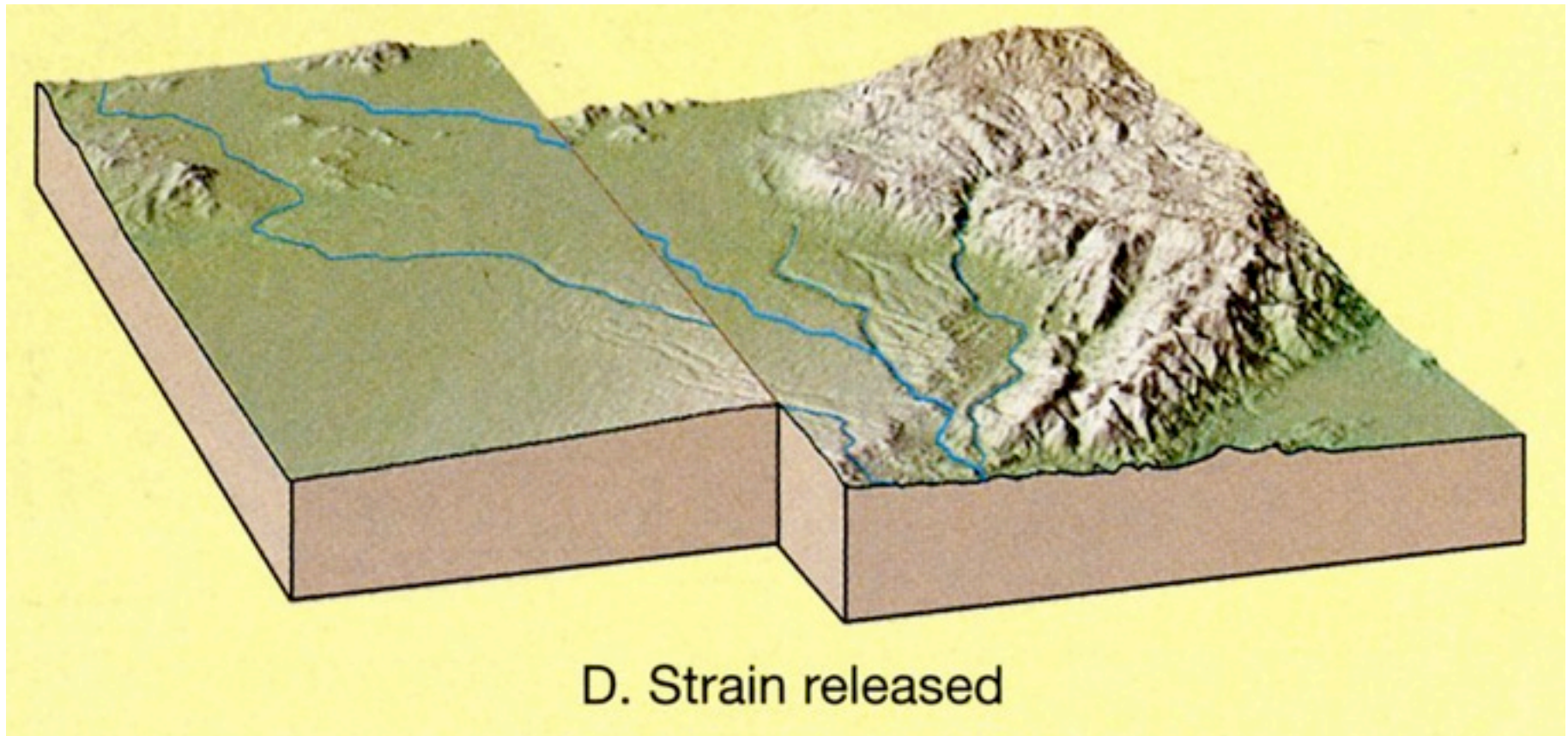




# Principle of the rebound hypothesis

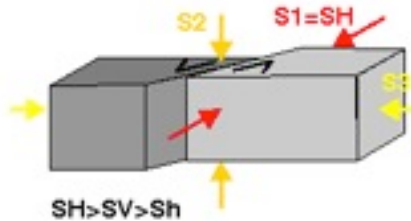


# Principle of the rebound hypothesis

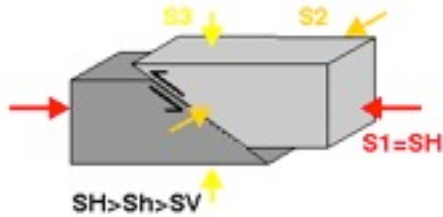
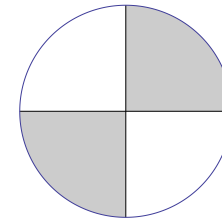


# Slip is not only horizontal

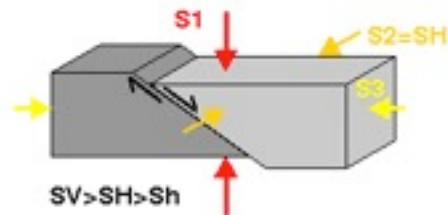
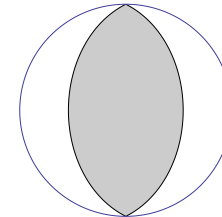
## Focal mechanism



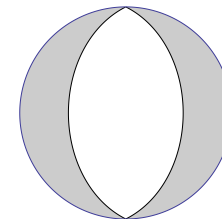
Strike slip



Reverse



Normal

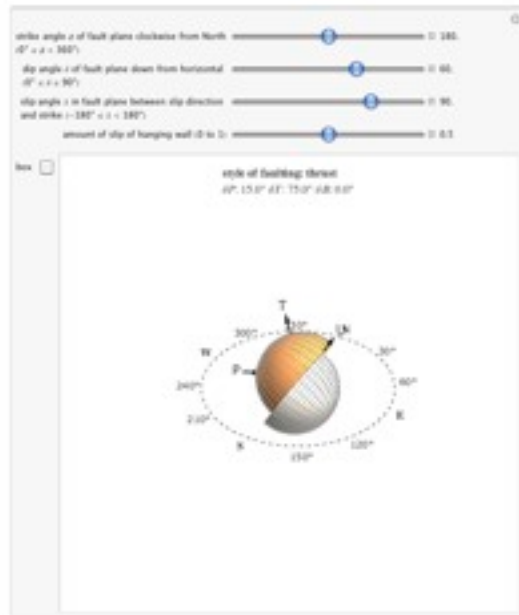


# Exercise

## StylesOfFaulting.cdf

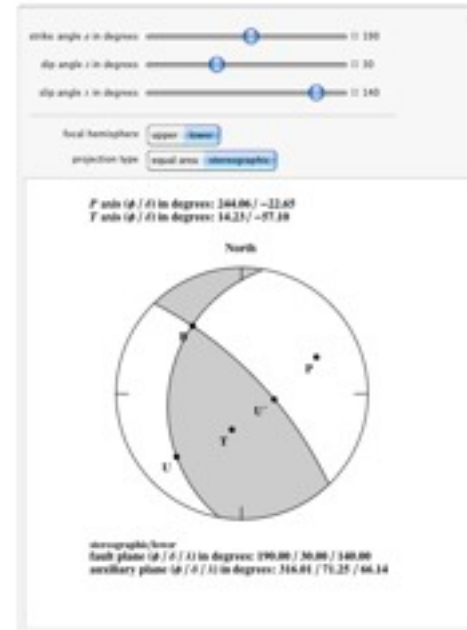
## EarthquakeFocalMechanism.cdf

### Styles of Faulting



This Demonstration provides a three-dimensional illustration of the different styles of faulting (e.g., during earthquakes, but also for slow deformations), as determined by the strike direction  $\phi$  of the fault: clockwise against geographic North, the dip angle  $\delta$  of the fault plane against the surface of the Earth, and the slip angle  $\alpha$  of the so-called hanging block (orange) against the foot wall block (white).

### Earthquake Focal Mechanism



This Demonstration generates earthquake focal mechanism plots, also called fault plane solution plots or simply beach balls. You can define the strike, dip, and rake angle of a fault and choose between different types of projections and focal hemispheres.

- What is the style of faulting of an earthquake in which the fault strikes with an angle of 60 degrees NE, dips with 55 degrees to the SE and in which the hanging wall slips at an angle of 45 degrees ( counterclockwise against the horizontal in strike direction)?
- Generate the focal mechanism and discuss the different sections.

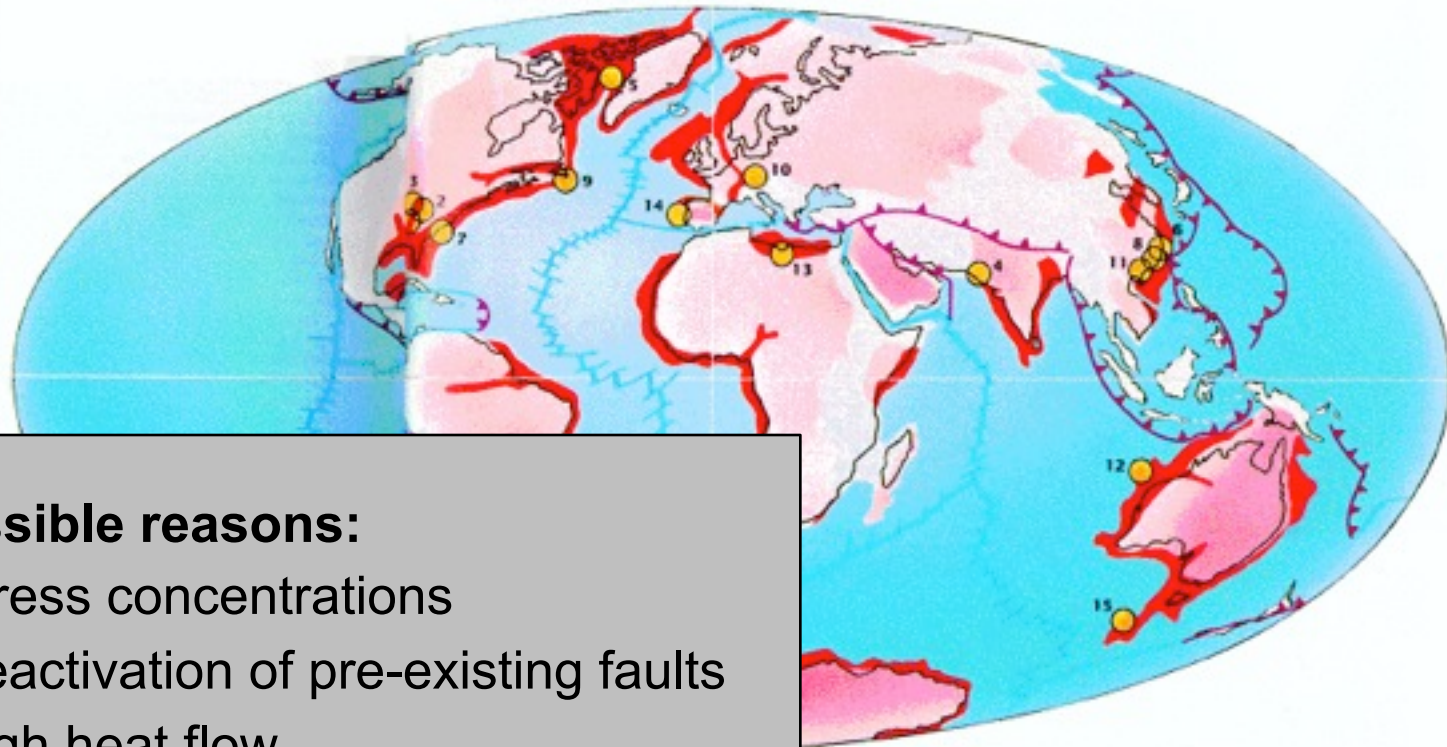


# Intraplate earthquakes

| Beben                       | Be-<br>reich | M   |
|-----------------------------|--------------|-----|
| 1. New Madrid 1812          | Rift         | 8,3 |
| 2. New Madrid 1811          | Rift         | 8,2 |
| 3. New Madrid 1812          | Rift         | 8,1 |
| 4. Kutch 1819               | Rift         | 7,8 |
| 5. Buffinbai 1933           | Rand         | 7,7 |
| 6. Formosastraße 1604       | Rand         | 7,7 |
| 7. South Carolina 1886      | Rand         | 7,6 |
| 8. Nanai 1918               | Rand         | 7,4 |
| 9. Grand Banks 1929         | Rand         | 7,4 |
| 10. Basel 1356              | Rift         | 7,4 |
| 11. Hainan 1605             | Rift         | 7,3 |
| 12. Exmouth Plateau<br>1906 | Rand         | 7,2 |
| 13. Libyen 1935             | Rand         | 7,1 |
| 14. Portugal 1858           | Rand         | 7,1 |
| 15. Tasmanische 1951        | Rand         | 7,0 |



Bild 2: Stabile Bereiche.  
Drittel der gesamten Kruste  
um. Nicht in ihre Defizite  
eingeschlossen haben die  
grenzen und weite, verstreute  
oder jünger Verformung.  
Erdbeben innerhalb stabiler  
Mikro-Magnituden-Skala  
sich alle dort ereignet, wo  
bis vor 250 Millionen Ja

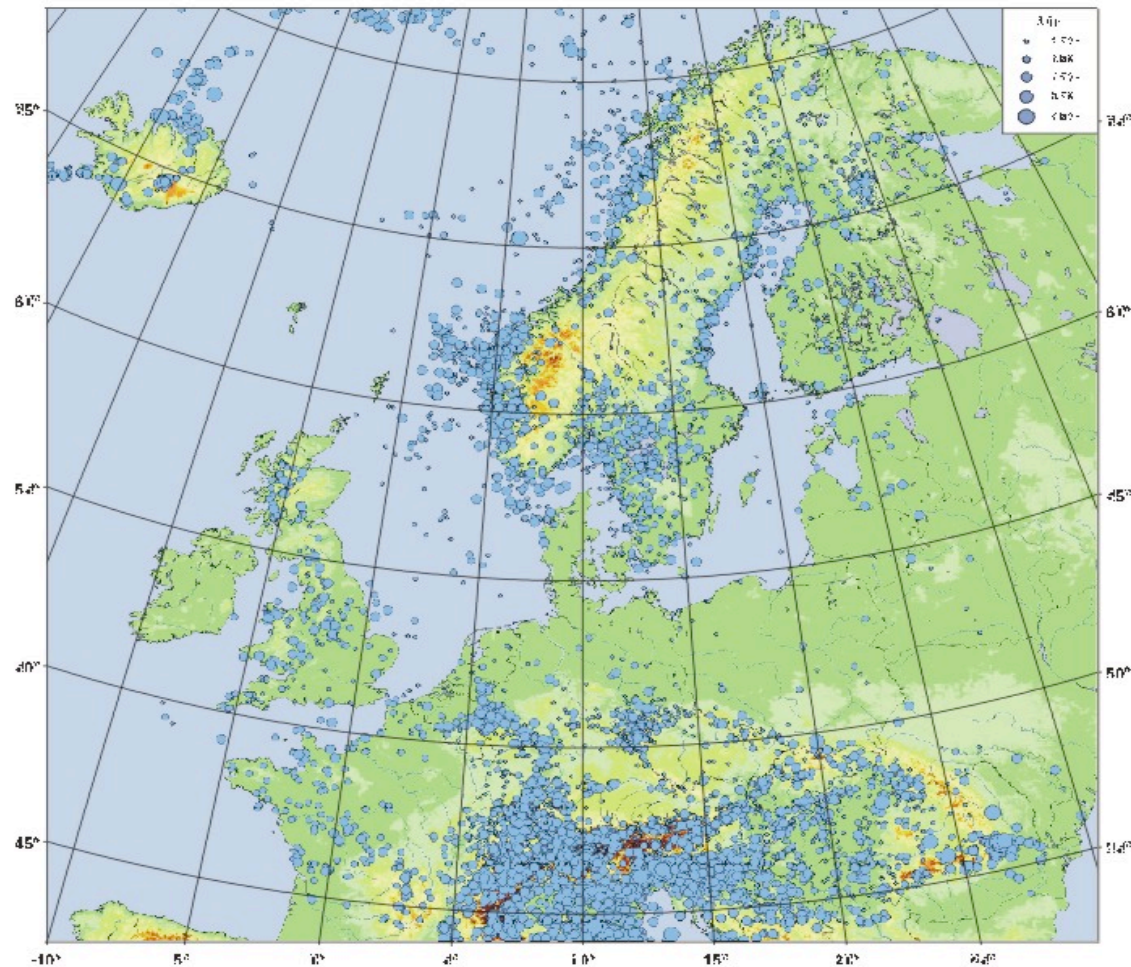


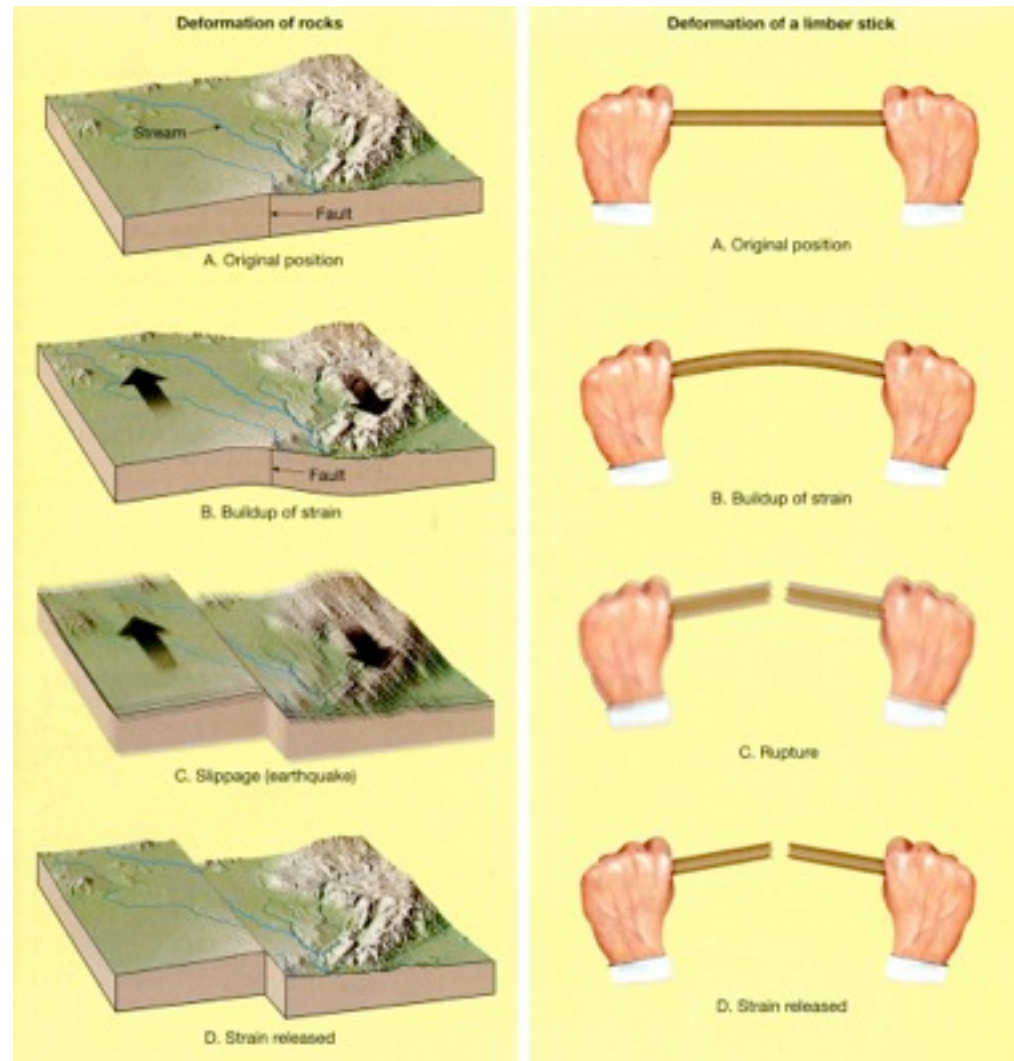
## Possible reasons:

- Stress concentrations
- Reactivation of pre-existing faults
- High heat flow

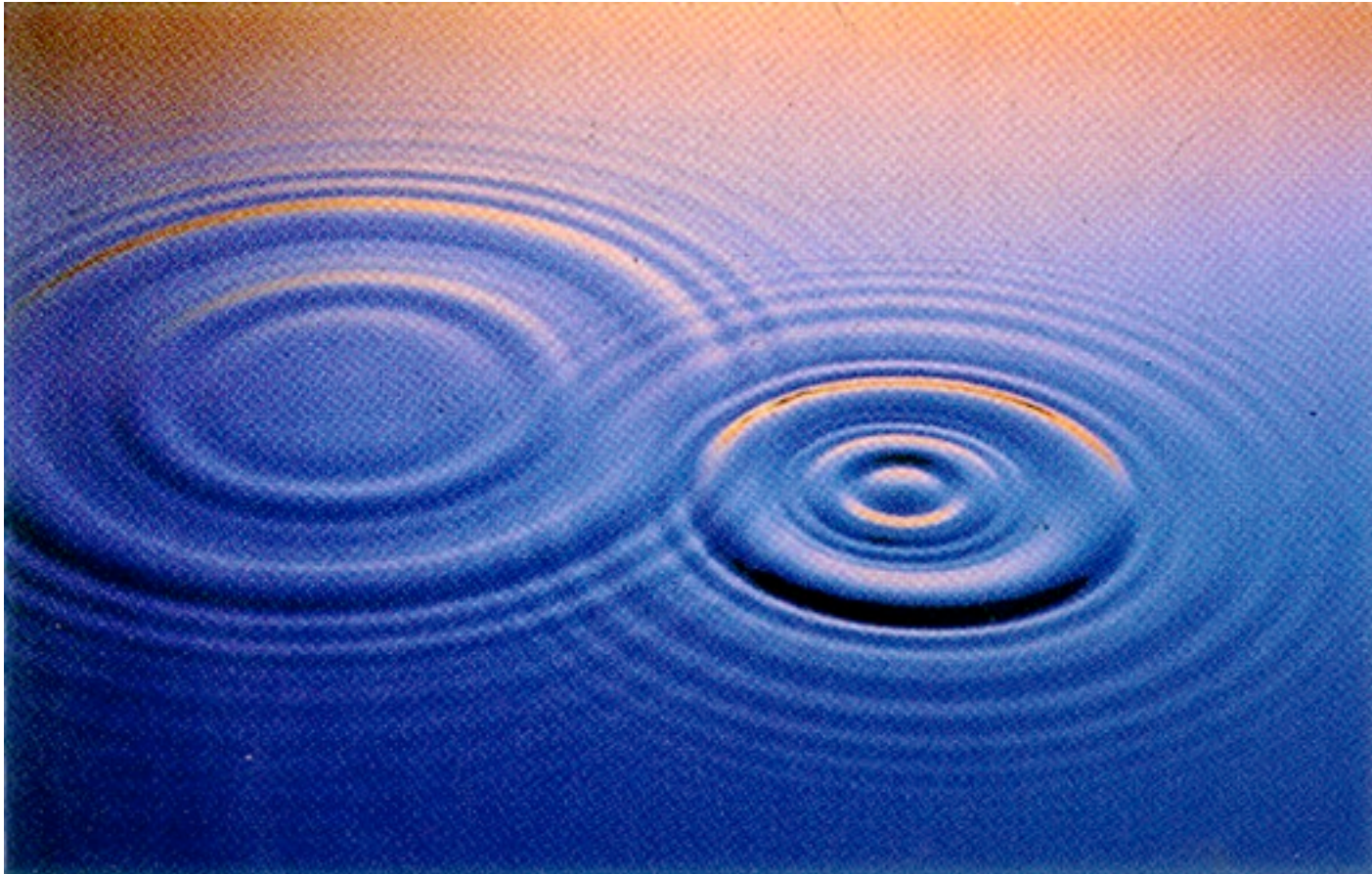
....

# Seismicity in Europe





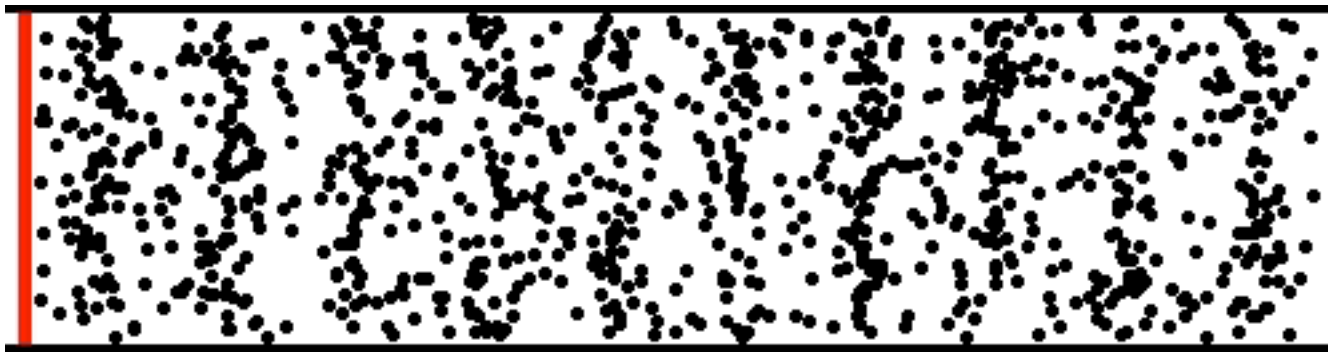




(E. Hecht, 1994)



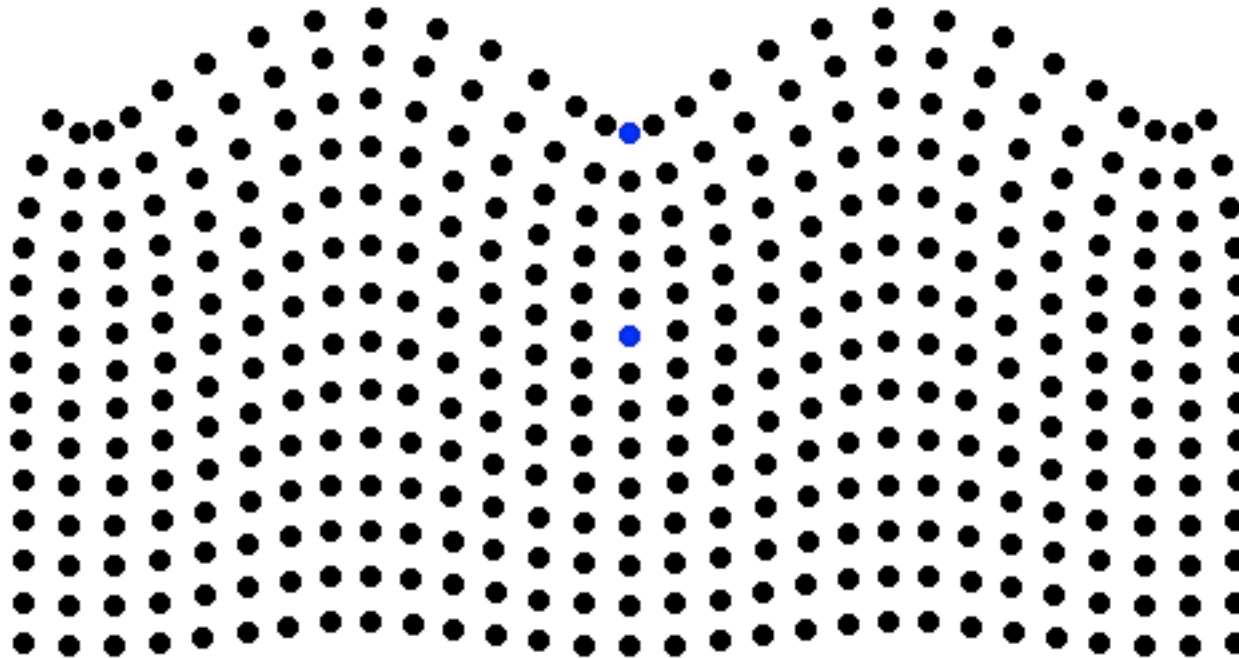
# P wave



# S wave

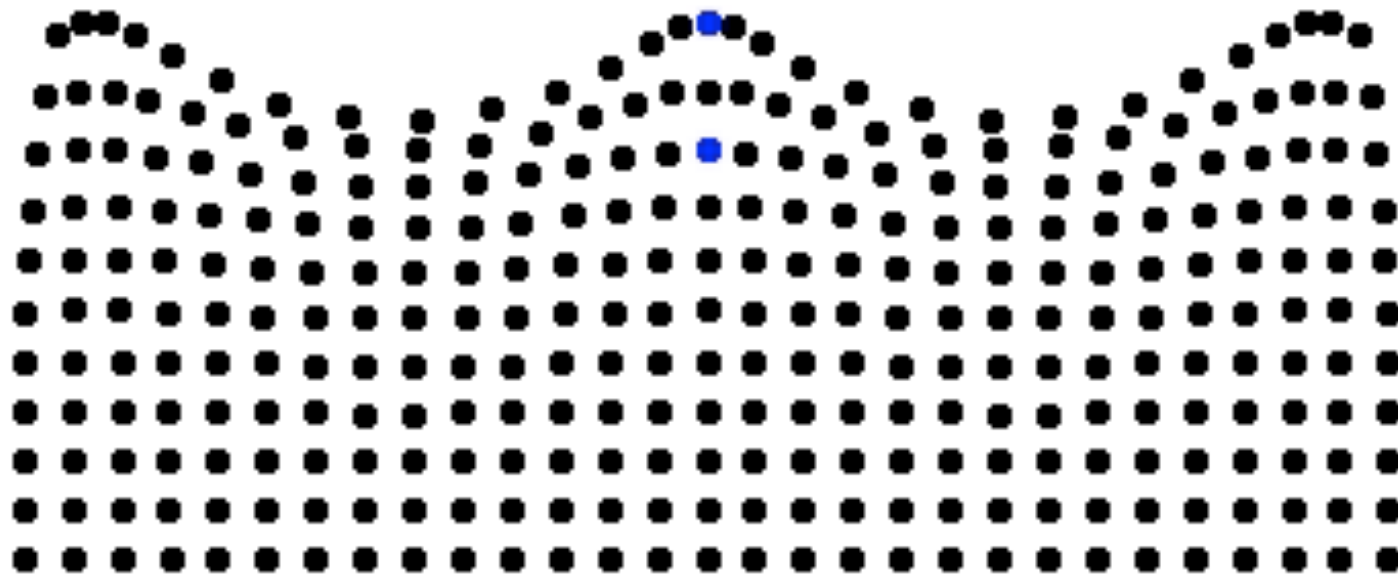


# Rayleigh wave



©1999, Daniel A. Russell

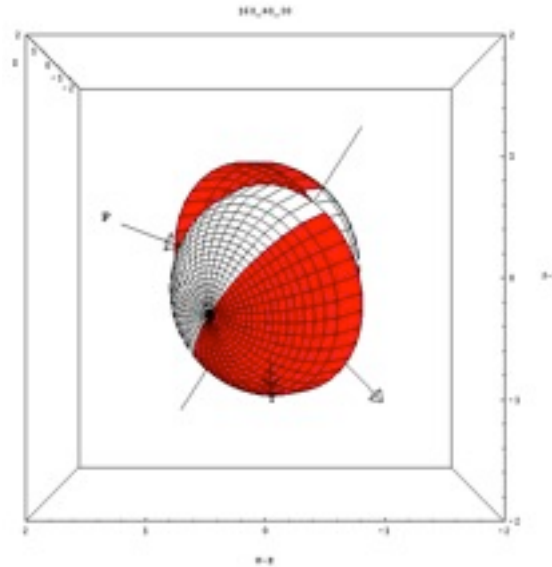
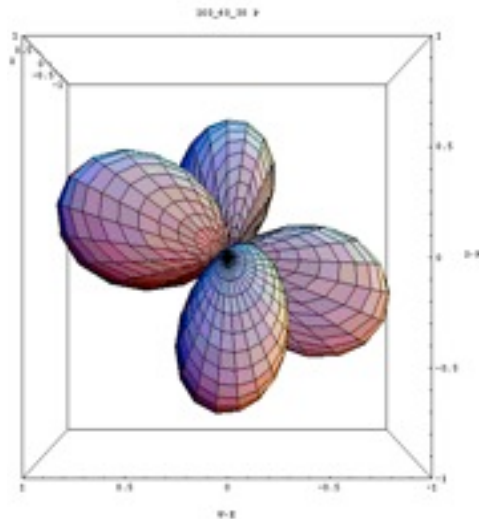
# Water wave



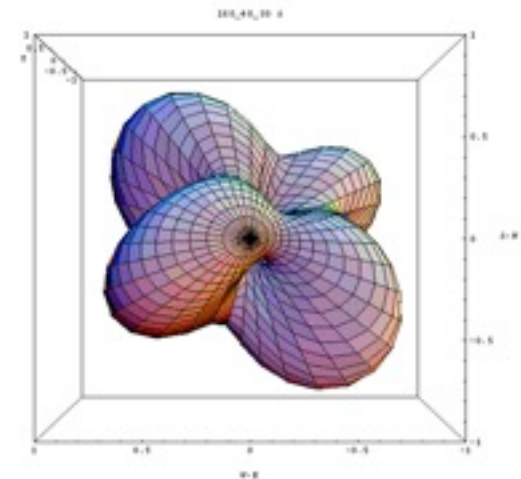
©1999, Daniel A. Russell

# Radiation from an earthquake source

P waves



S waves



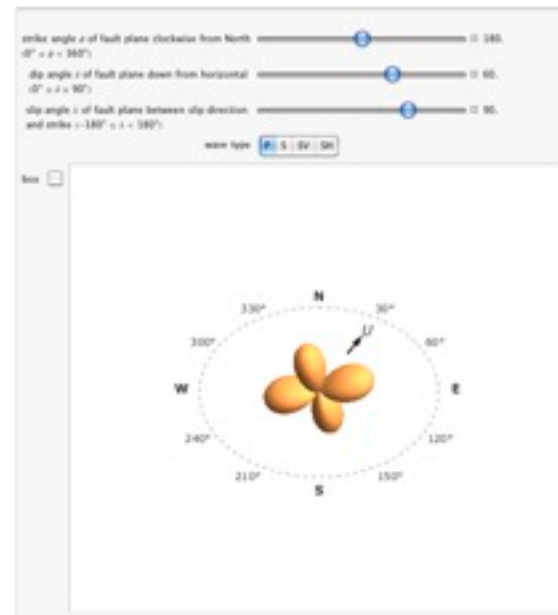
# Exercise

## DeformationPatternInAnEarthquakeSourceRegion.cdf RadiationPatternForDoubleCoupleEarthquakeSources.cdf

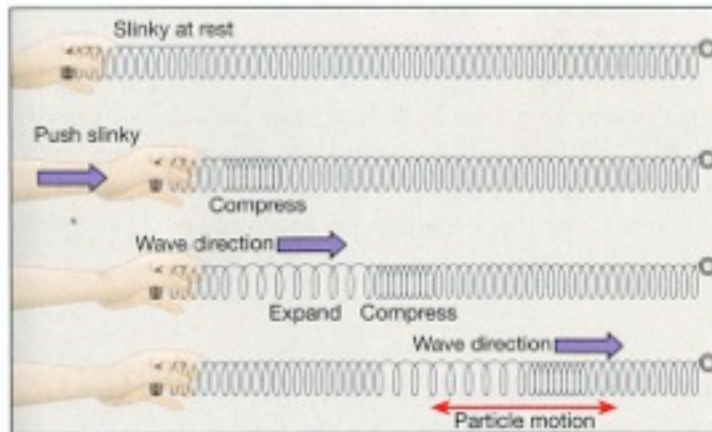
Deformation Pattern in an Earthquake Source Region



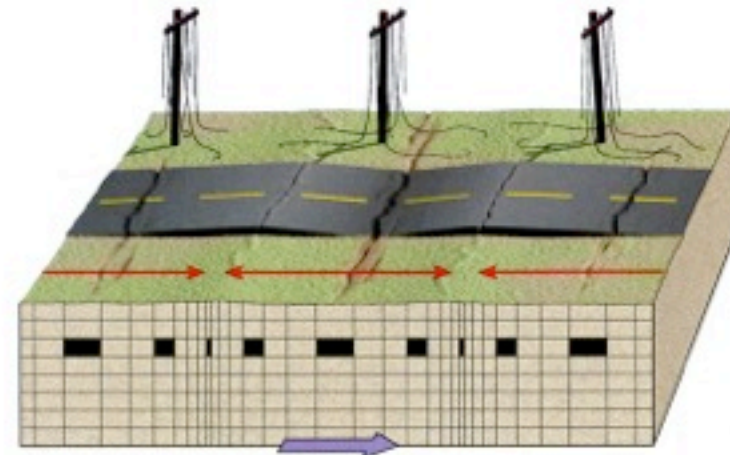
Radiation Pattern for Double-Couple Earthquake Sources



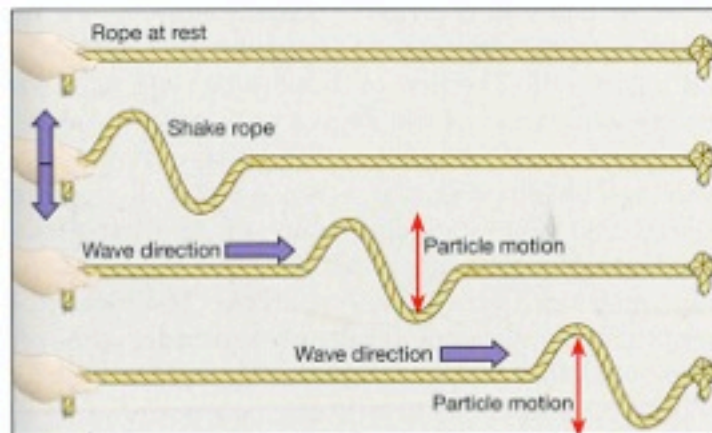
- What is the deformation pattern of an earthquake in which the fault strikes with an angle of 60 degrees NE, dips with 55 degrees to the SE and in which the hanging wall slips at an angle of 45 degrees (counterclockwise against the horizontal in strike direction)?
- Generate the corresponding radiation pattern and discuss the different sections.



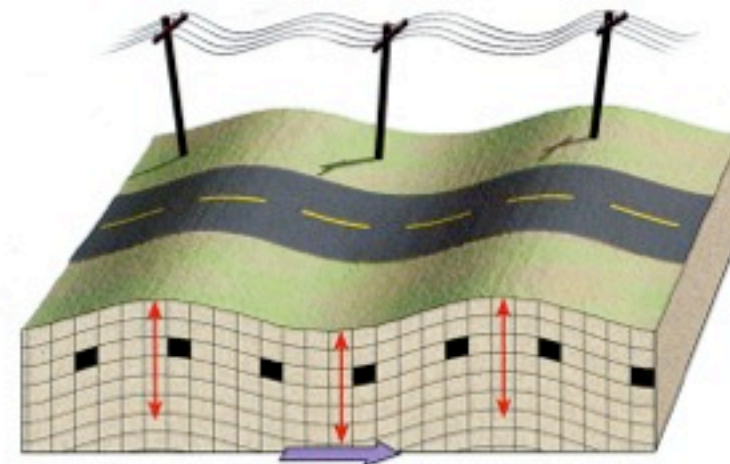
A. P waves generated using a slinky



B. P waves traveling along the surface



C. S waves generated using a rope



D. S waves traveling along the surface







# Loma Prieta, 1989



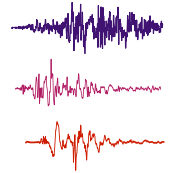
# Mexico City, 1985





# Earthquake ground motion in a nutshell

Source → Crustal propagation → Site effects →



- rupture propagation effects (directivity)
- source heterogeneity effects (asperities, barriers)
- source extension effects (interference)
- hanging wall /foot wall

- geometrical spreading,
- scattering
- anelastic attenuation
- reflection (e. g. Moho bounce),
- refraction
- focusing, defocusing
- 

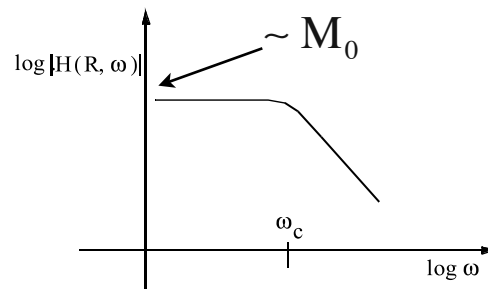
- reverbaration
- basin effects
- non-linear damping
- kappa- effect
-

## Seismic moment $M_0$

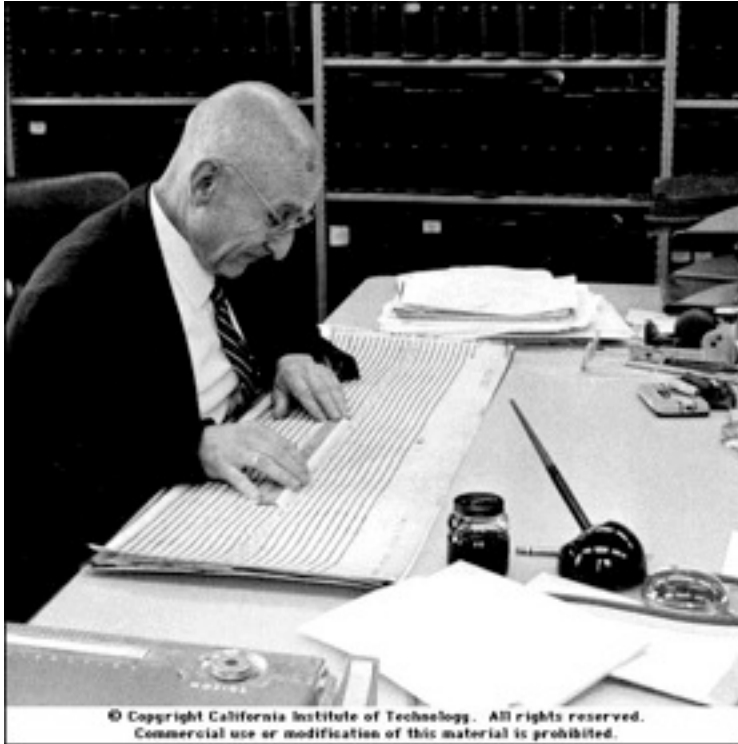
$$M_0 = \mu \cdot d_0 \cdot A \quad [\text{N m}]$$

shear modulus  $[\text{N/m}^2]$       mean dislocation  $[\text{m}]$       rupture plane area  $[\text{m}^2]$

Moment determination e. g.:



# Earthquake strength: magnitudes

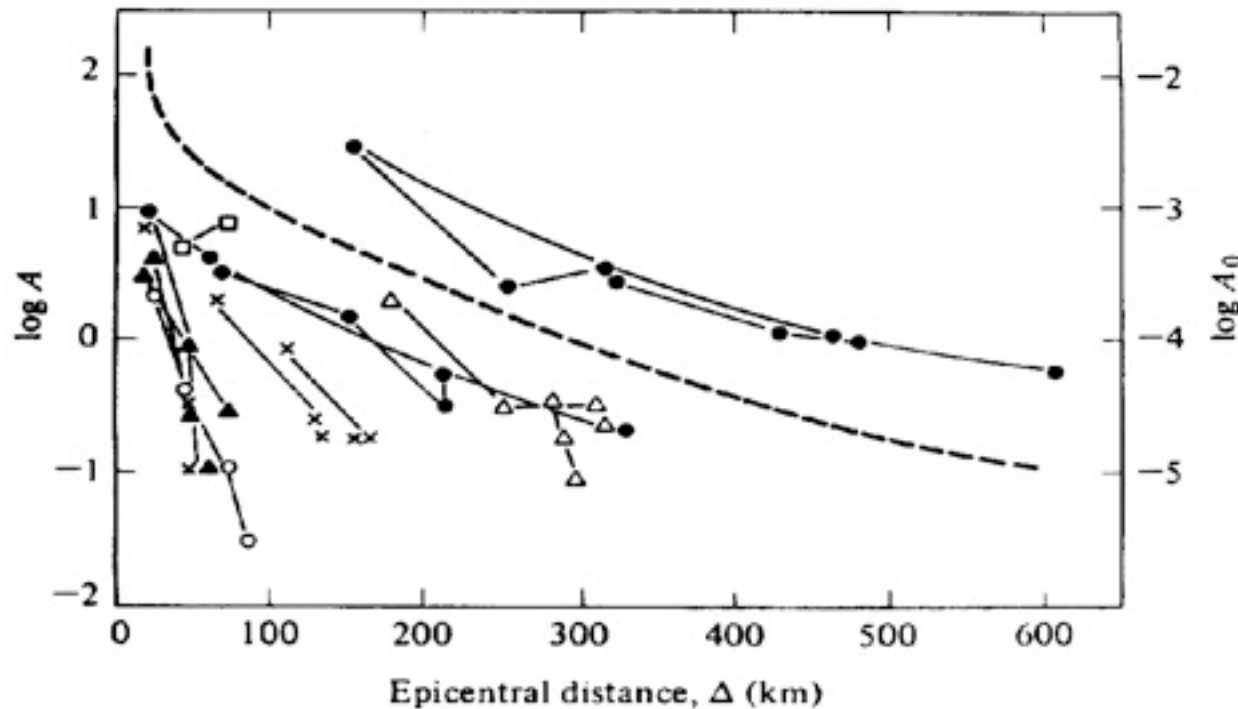


B. Gutenberg



C. F. Richter

# Magnitude



Amplitude -  
distance relations  
for Southern  
California  
earthquakes

- Amplitude ratios widely independent of measurement site

# First magnitude definition 1935, C.F. Richter

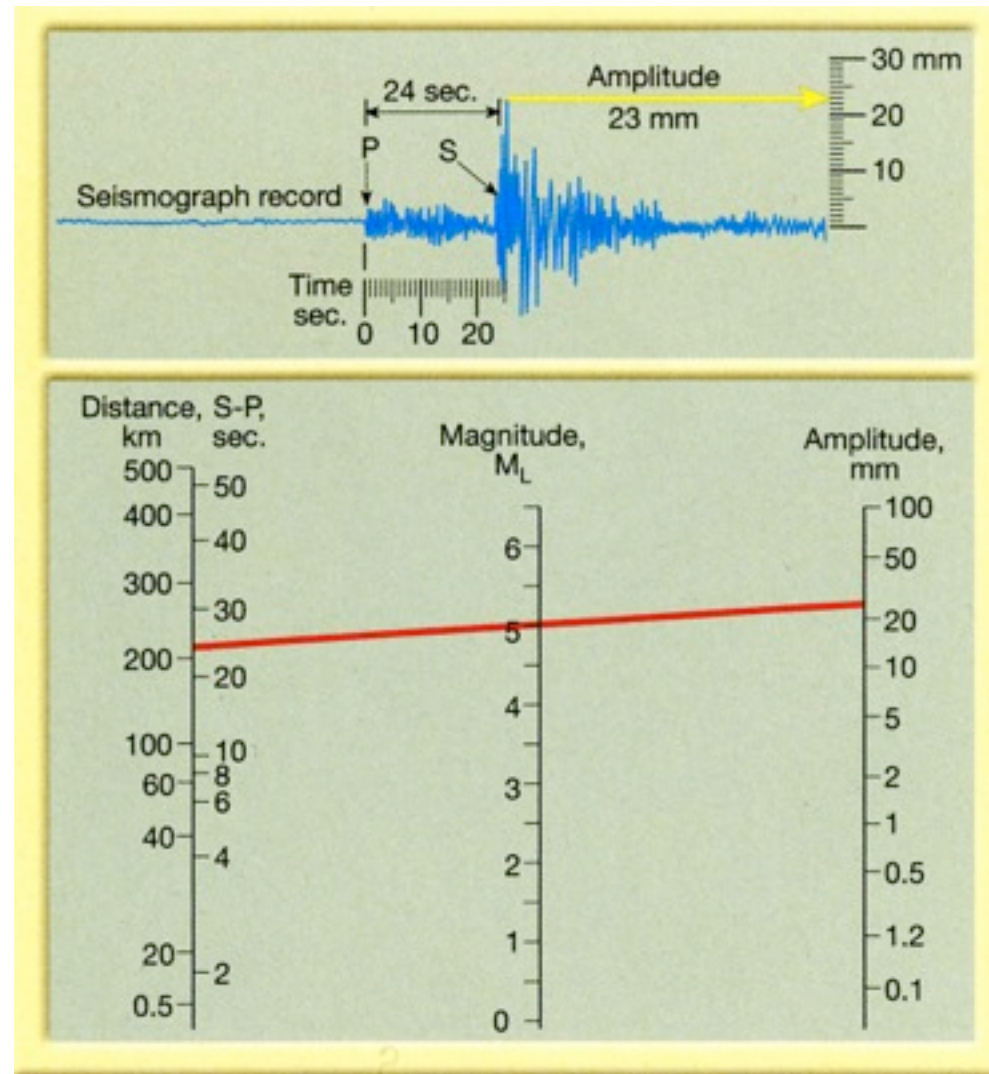
## „Richter-, Wood-Anderson-, local magnitude“

- Amplitude measurement on Wood Anderson displacement seismometer ( $T_0 = 0.8$  s,  $h = 0.8$ ,  $V = 2800$ )
- Comparison to expected amplitude of reference earthquake

$$M_{WA} = \log A(\Delta) - \log A_{ref}(\Delta)$$

- $\Delta$  = epicentral distance
- $\log A_{ref}(\Delta)$  = amplitude of  $M_{WA} = 0$  quake at the site
- Definition of reference amplitude: half PP-amplitude of  $M_{WA} = 3$  quake at 100 km distance is 1 mm

# Graphical Richter magnitude determination





# Other magnitude definitions



**Body wave magnitude  $M_b$**

**Surface wave magnitude  $M_s$**

**Moment magnitude**  $M_w = \frac{\log 10(M_0)}{1,5} - 10,73$

$\sim$  Energy  
↙

$$\rightarrow \Delta M_w = 1 \Rightarrow \Delta \log 10(E) = 1.5 \Rightarrow \text{Energy ratio} = 10^{1.5} \approx 32$$

- Most magnitude scales not valid for all distance/depth ranges
- Other problems e. g.: saturation, determination effort
- In the context of SHA:  $M_w$  least problematic (no saturation)

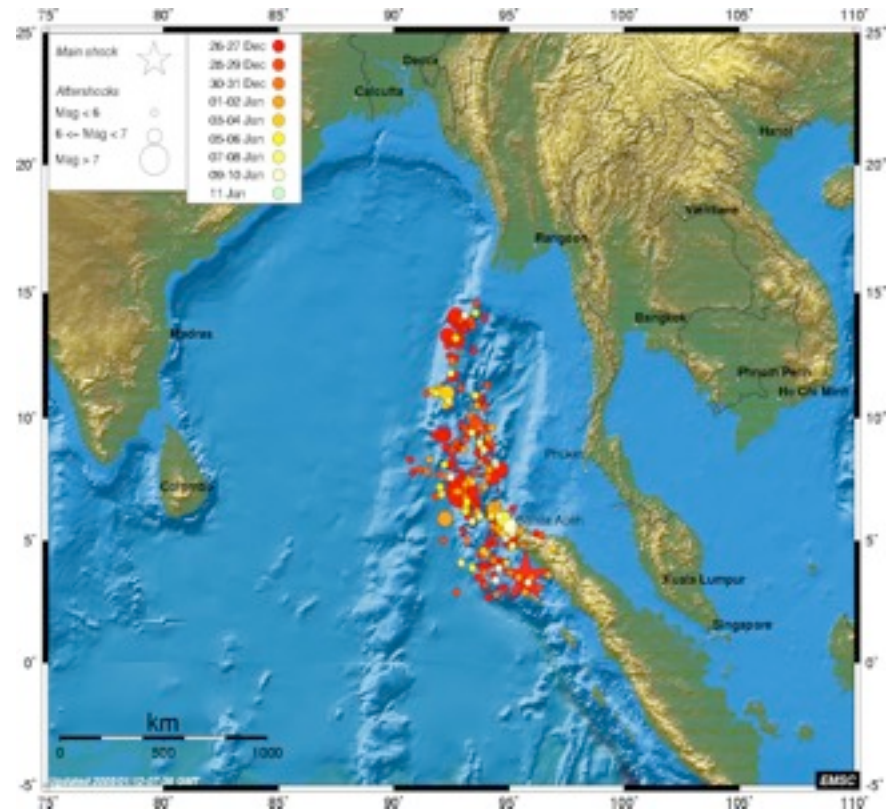
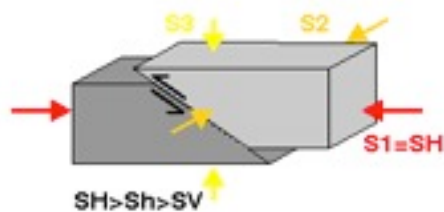
# Some dimensions

| Magnitude $M_w$ | Length (km)   |
|-----------------|---------------|
| 4               | $< 1$         |
| 5               | $\approx 2$   |
| 6               | $\approx 10$  |
| 7               | $\approx 50$  |
| 8               | $\approx 250$ |

Take with many grains of salt.....

# Example: Sumatra earthquake

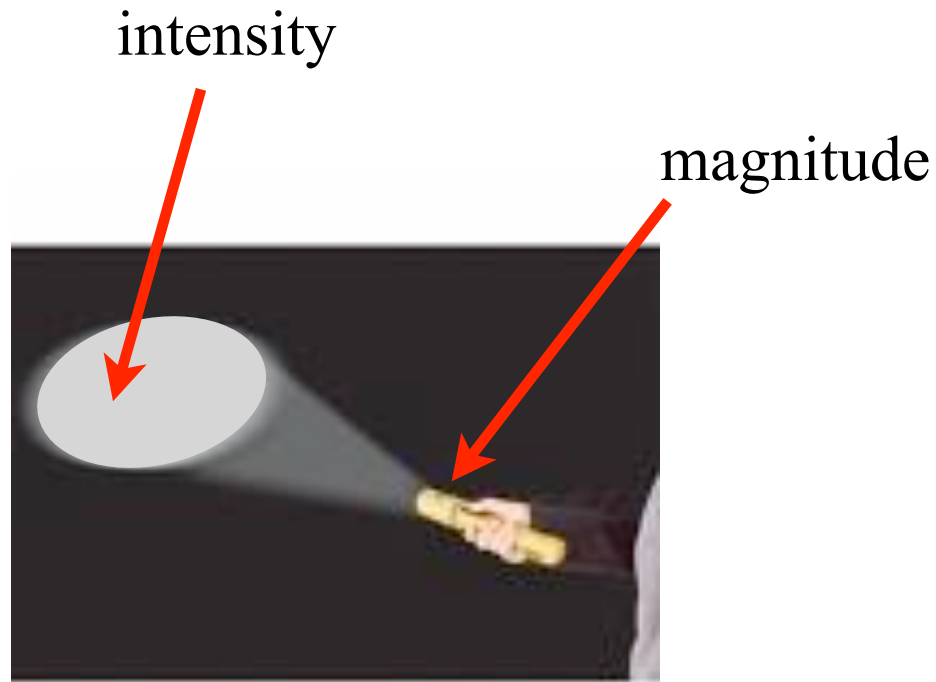
- **26.12.2004, 00:58:53 UTC, MW 9**
- **Rupture length > 1000 km**
- **Displacement up to 20 m**
- **Energy: 3 month Europe**



**TABLE 7.1** Modified Mercalli Intensity Scale.

- I Not felt except by a very few under especially favorable circumstances.
- II Felt only by a few persons at rest, especially on upper floors of buildings.
- III Felt quite noticeably indoors, especially on upper floors of buildings, but many people do not recognize it as an earthquake.
- IV During the day felt indoors by many, outdoors by few. Sensation like heavy truck striking building.
- V Felt by nearly everyone, many awakened. Disturbances of trees, poles, and other tall objects sometimes noticed.
- VI Felt by all; many frightened and run outdoors. Some heavy furniture moved; few instances of fallen plaster or damaged chimneys. Damage slight.
- VII Everybody runs outdoors. Damage negligible in buildings of good design and construction; slight-to-moderate in well-built ordinary structures; considerable in poorly built or badly designed structures.
- VIII Damage slight in specially designed structures; considerable in ordinary substantial buildings with partial collapse; great in poorly built structures. (Fall of chimneys, factory stacks, columns, monuments, walls.)
- IX Damage considerable in specially designed structures. Buildings shifted off foundations. Ground cracked conspicuously.
- X Some well-built wooden structures destroyed. Most masonry and frame structures destroyed. Ground badly cracked.
- XI Few, if any, (masonry) structures remain standing. Bridges destroyed. Broad fissures in ground.
- XII Damage total. Waves seen on ground surfaces. Objects thrown upward into air.

# Differences between magnitude and intensity





# CDFs related to the seismology part

**UnderstandingEarthquakes.cdf (Wolfram Research CDF Demo)**

**EarthquakeFocalMechanism.cdf**

**DeformationPatternInAnEarthquakeSourceRegion.cdf**

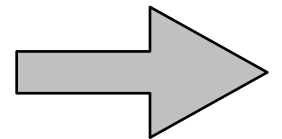
**RadiationPatternForDoubleCoupleEarthquakeSources.cdf**

**StylesOfFaulting.cdf**

**VerticalPendulumSeismometer.cdf**



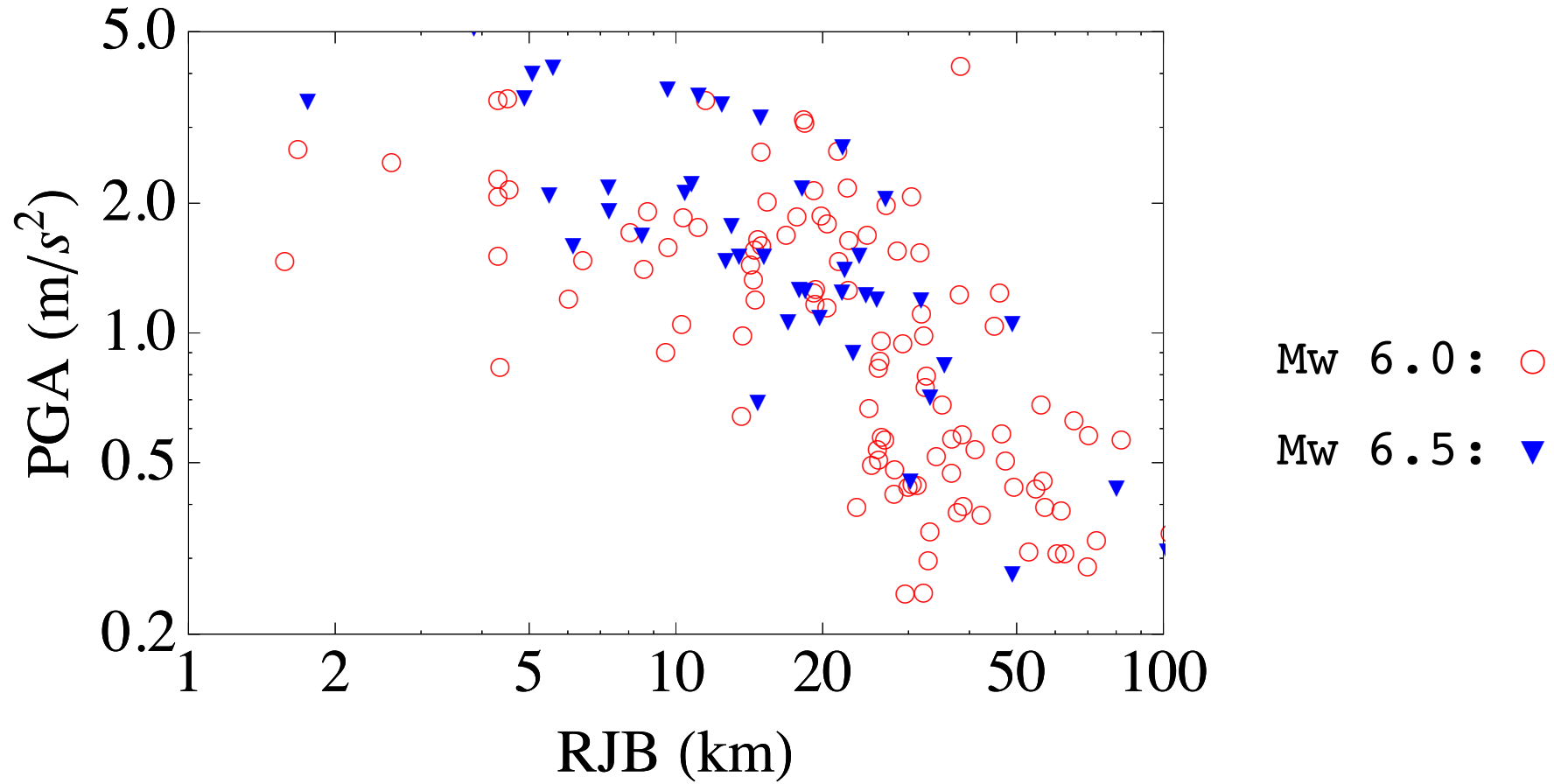
**For now enough seismology,  
back on the main road**



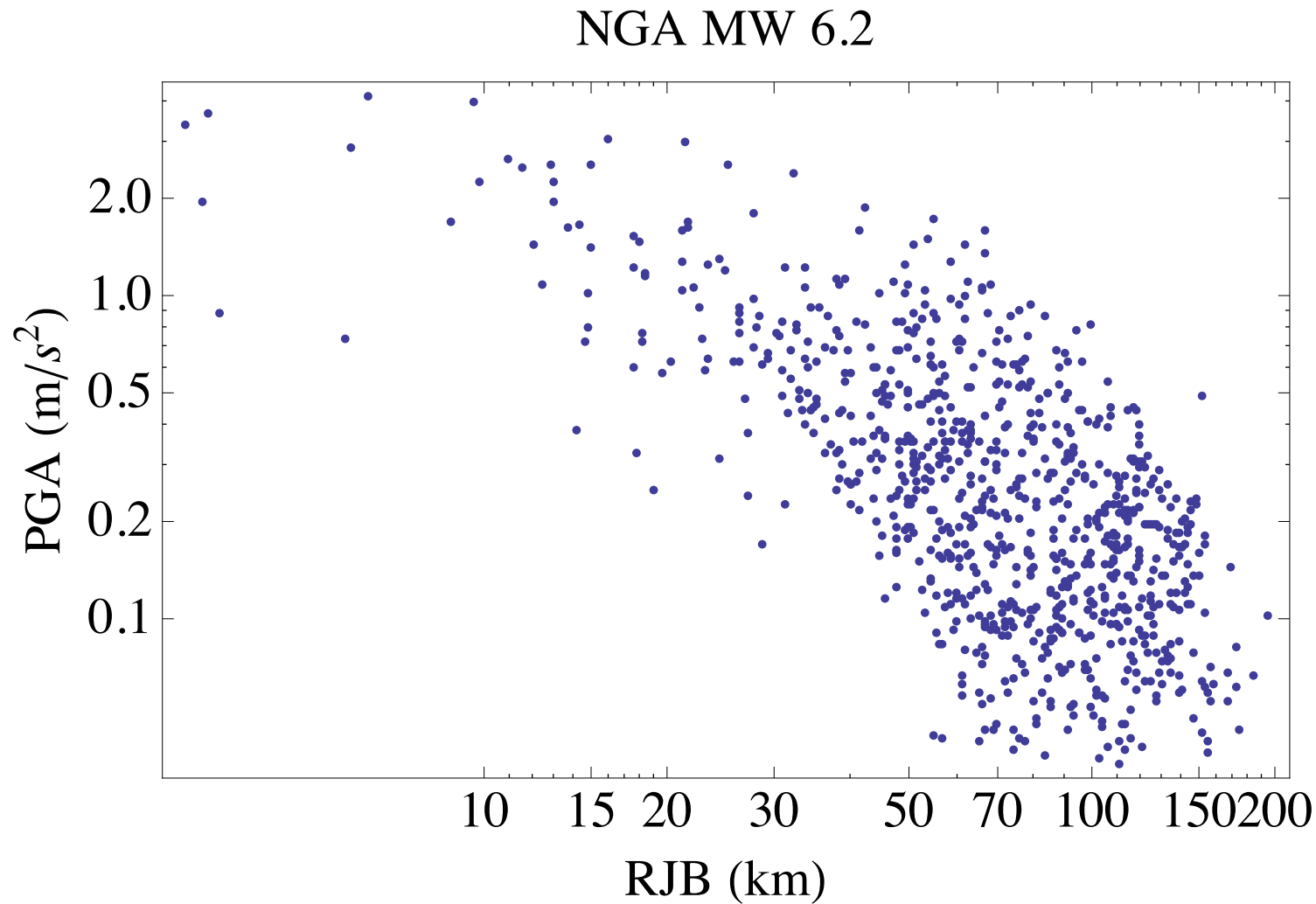
## Options

- complete models (e. g. 3D spectral elements)
- simplified models (e. g. stochastic models)
- **empirical regression models (GMPE)**

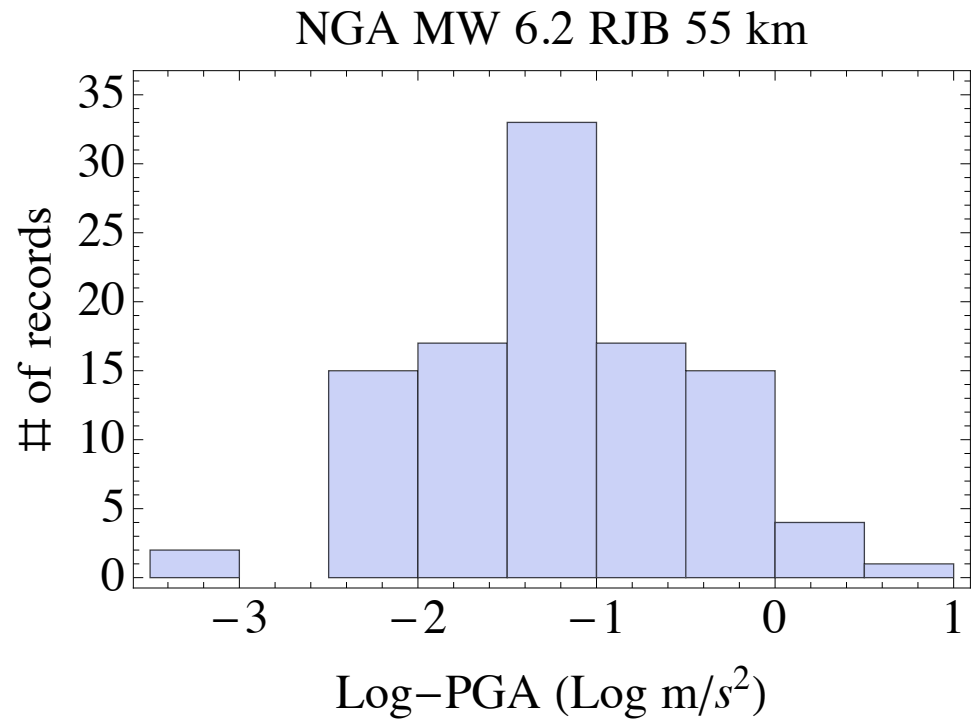
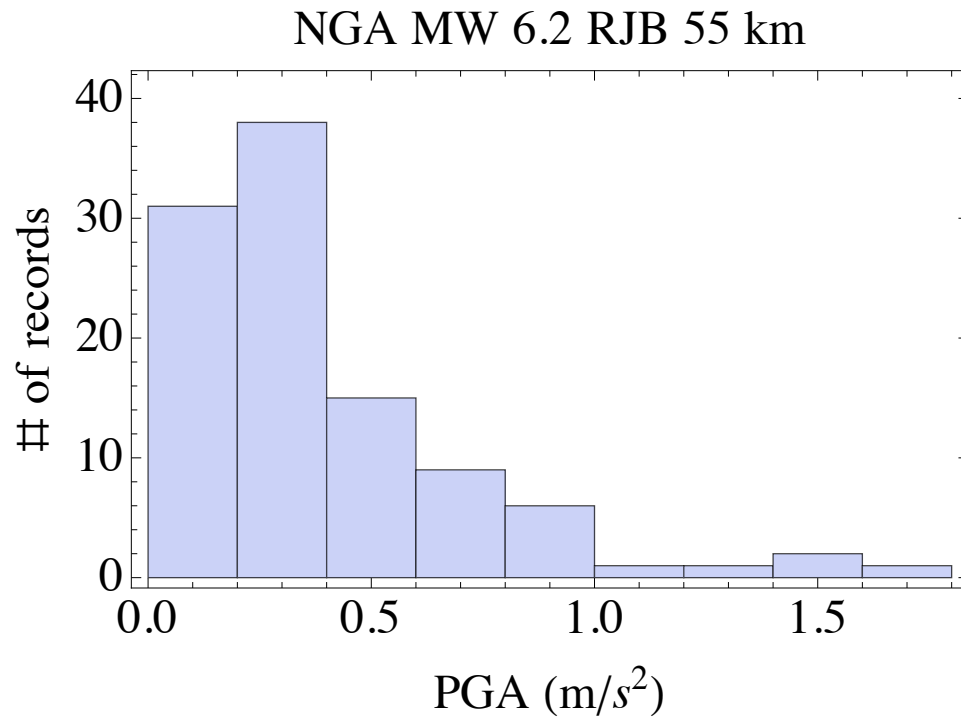
# Empirical data (NGA)



# Distance dependence



# PGA distribution

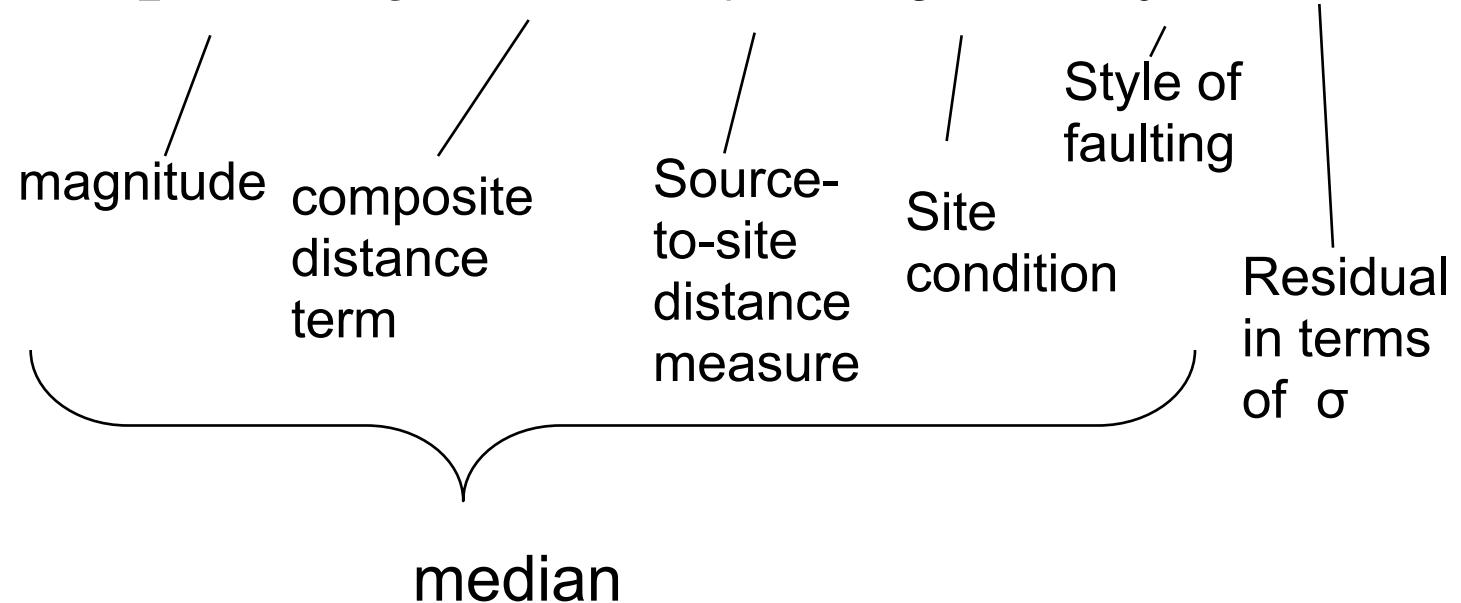




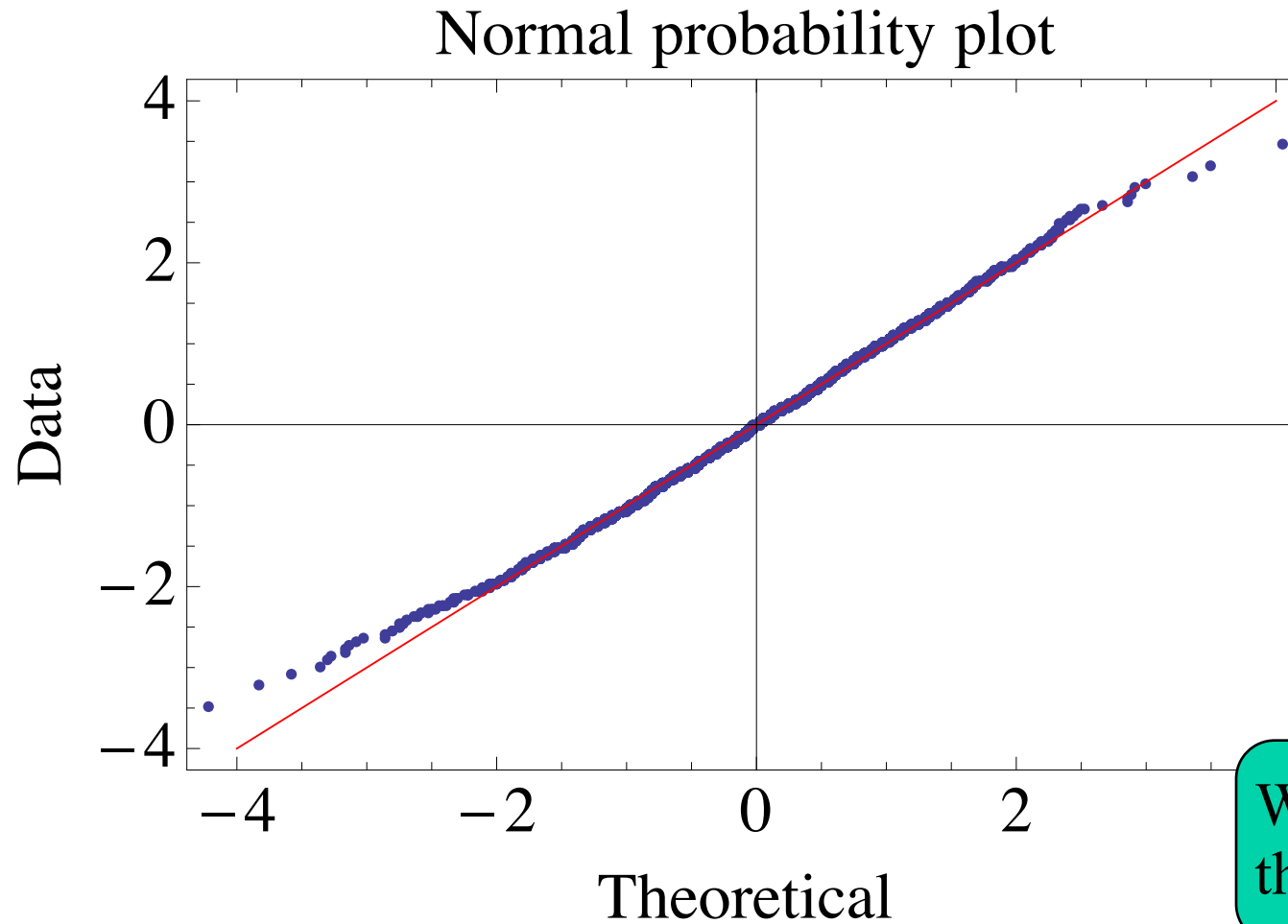
## Probabilistic model

Typical form:  $Y = c_1 e^{c_2 M} R^{-c_3} e^{-c_4 r} e^{-c_5 F} e^{-c_6 S} e^{\varepsilon \cdot \sigma}$

or:  $\ln Y = c_1 + c_2 M - c_3 \ln R - c_4 r + c_5 F + c_6 S + \varepsilon \cdot \sigma$



# InY-residual distribution NGA data



What causes  
the residuals ?

# One contribution: variability in the earthquake process

Example: What's represented by magnitude ( $M_w$ )...

$$M_w = 2/3 \cdot (\log M_0 - 9.1)$$

$$M_0 = \mu \cdot d_0 \cdot A$$

Shear modulus      Dislocation      area

**Mw 5.5:**

| Radius [km] | Dislocation [cm] |
|-------------|------------------|
| 8.4         | 3                |
| 6.5         | 5                |
| 4.6         | 10               |
| ....        | ....             |

- relative dislocation controls stress drop
- stress drop controls HF ground motion
- **earthquakes of same magnitude can produce very different ground motion**

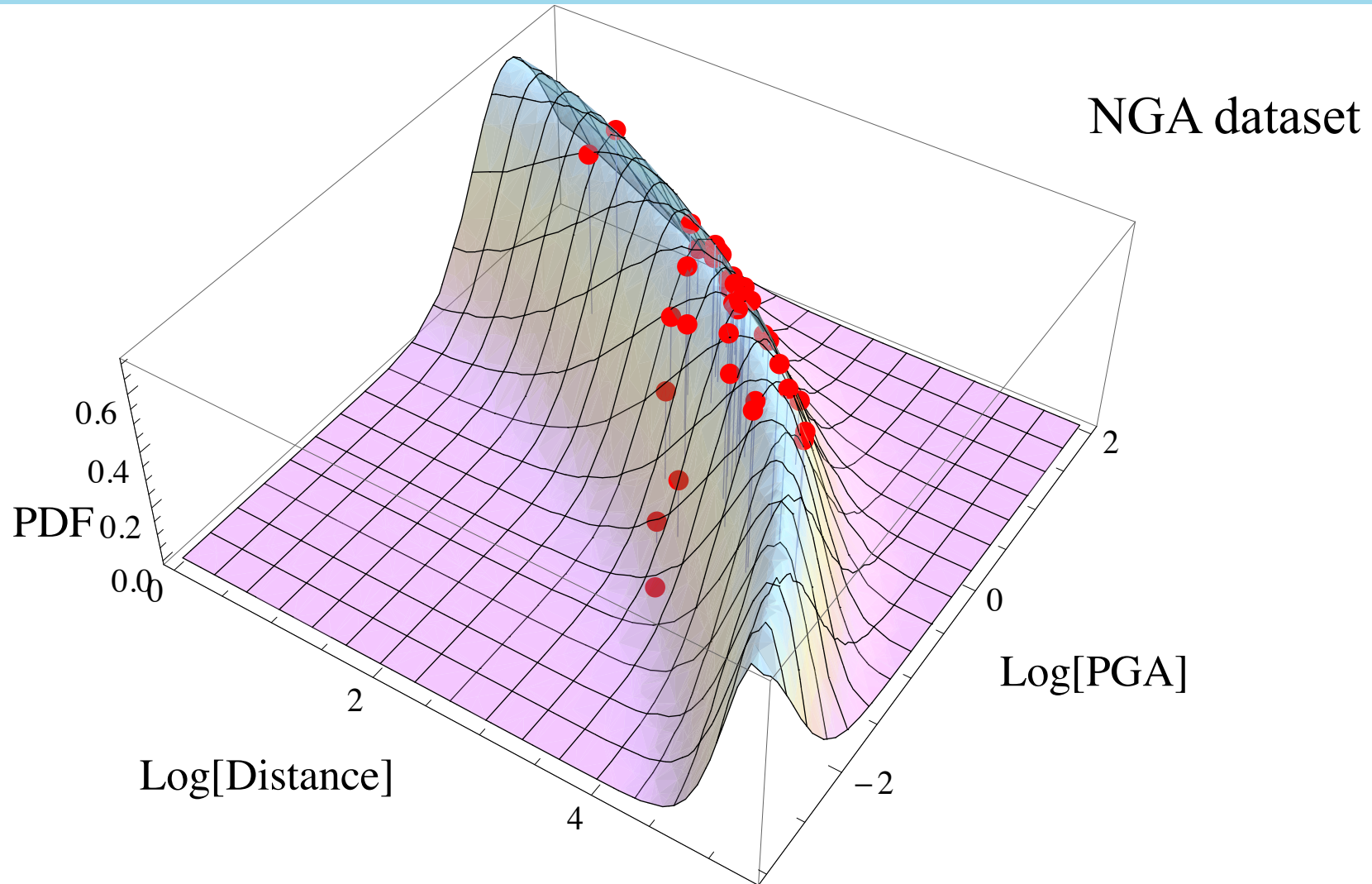
# Consequences



For an earthquake of a particular magnitude in a particular distance at a particular site:

- there is no single „true“ ground motion value for an EQ of fixed magnitude at fixed distance !
- **ground-motion must be treated as random variable** reflecting the intrinsic randomness of the earthquake process (aleatory uncertainty).
- assumption (based on data) log-normal distribution

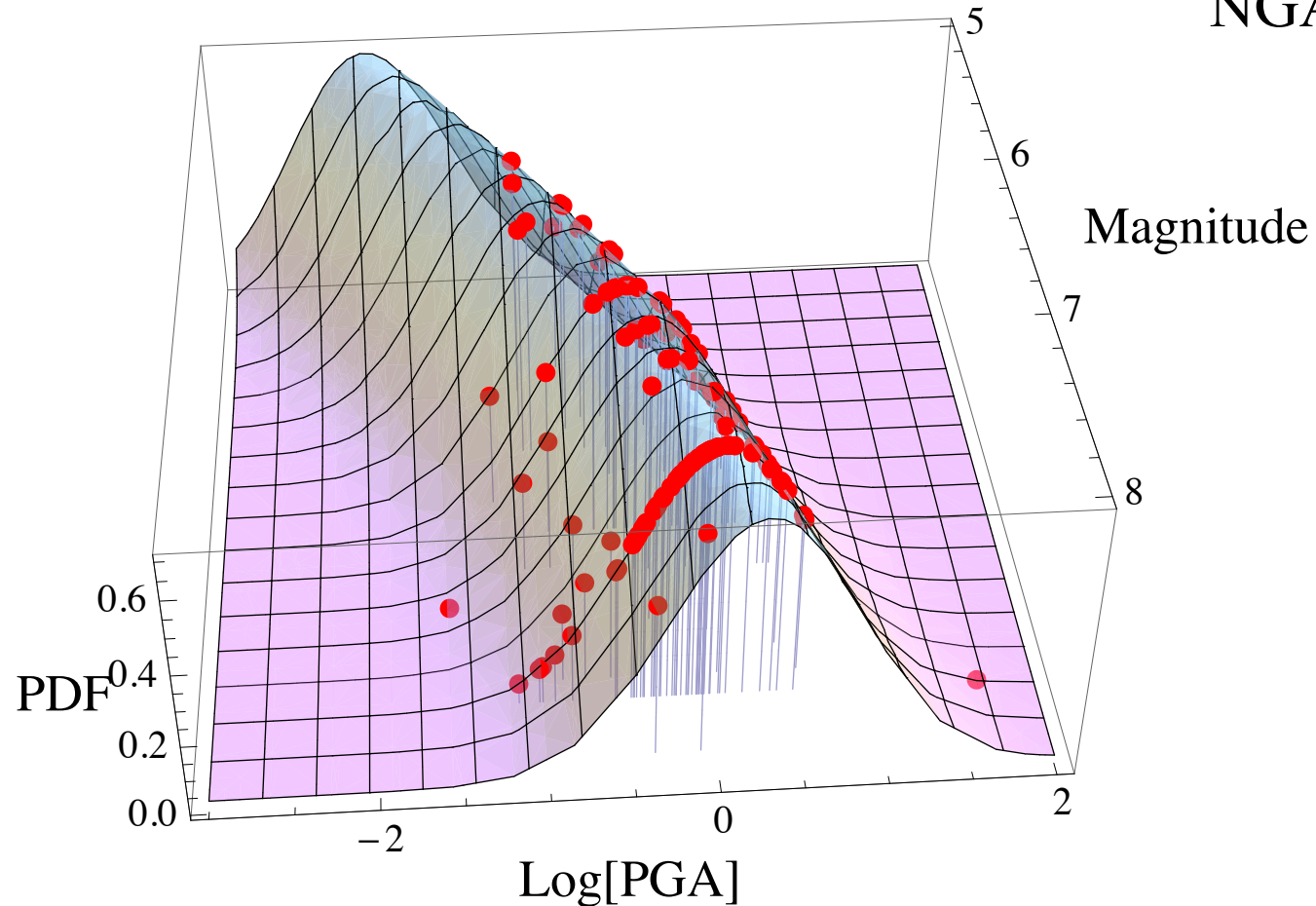
# Distance scaling: Mw 6.2



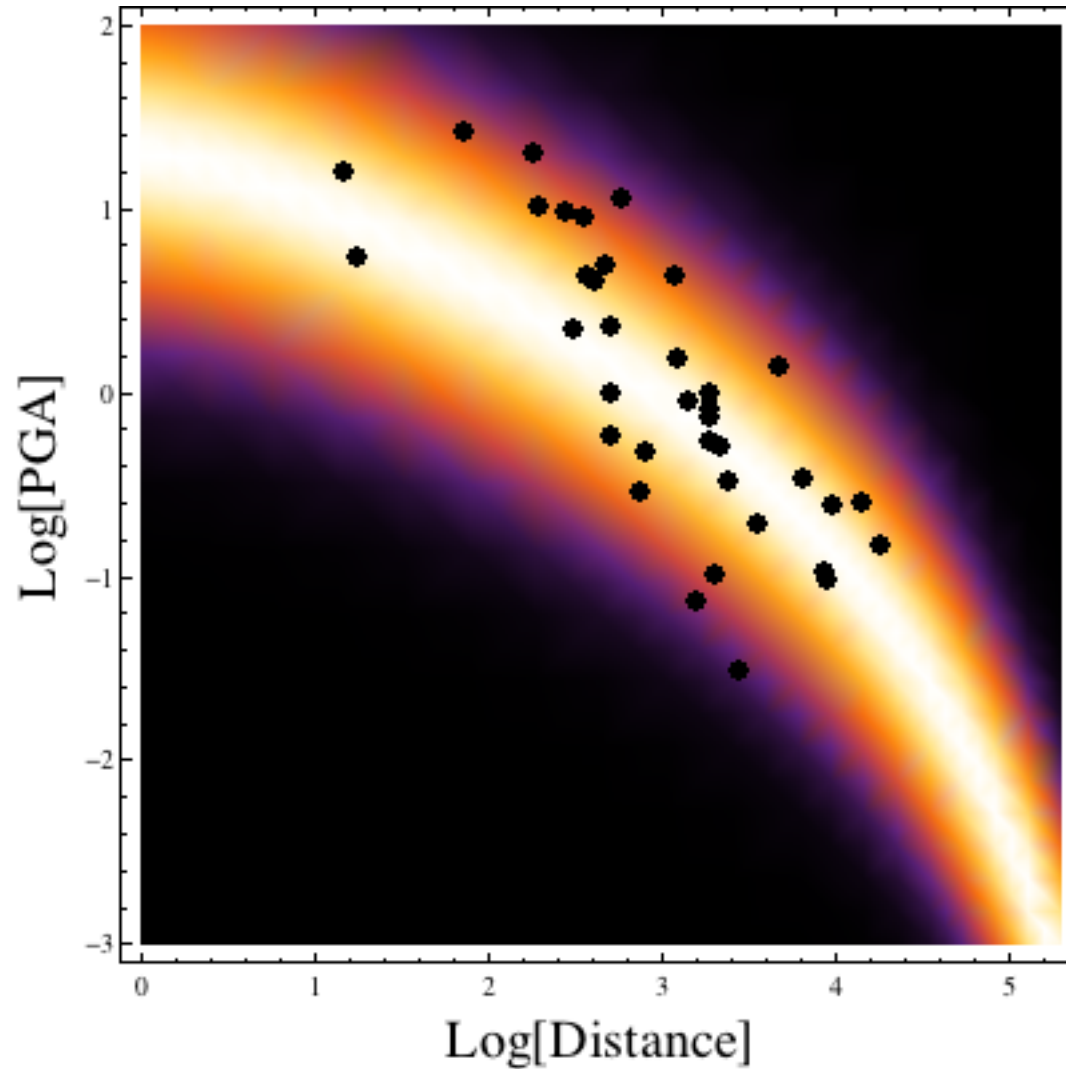


# Magnitude scaling: RJB 45-55 km

NGA dataset



# GMPEs are probabilistic models!!!



NGA dataset

Mw 6.2

# Single earthquake source

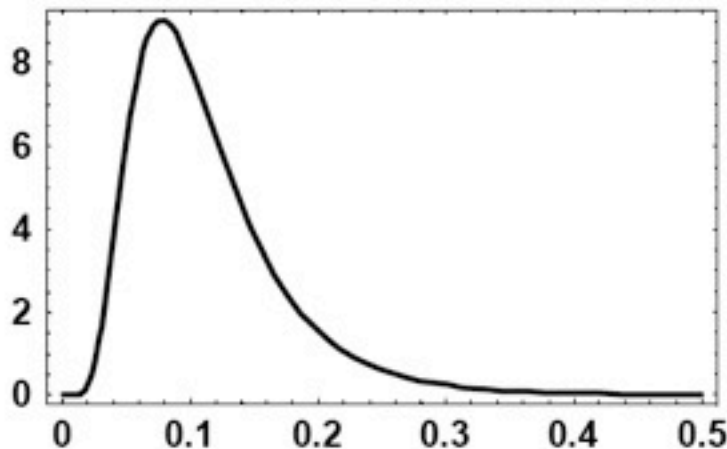
# Single source producing variable ground motion levels (RV)

Let us assume  $a$  is log-normally distributed  $f(x) = \frac{1}{\sigma \sqrt{2\pi} \cdot x} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}$

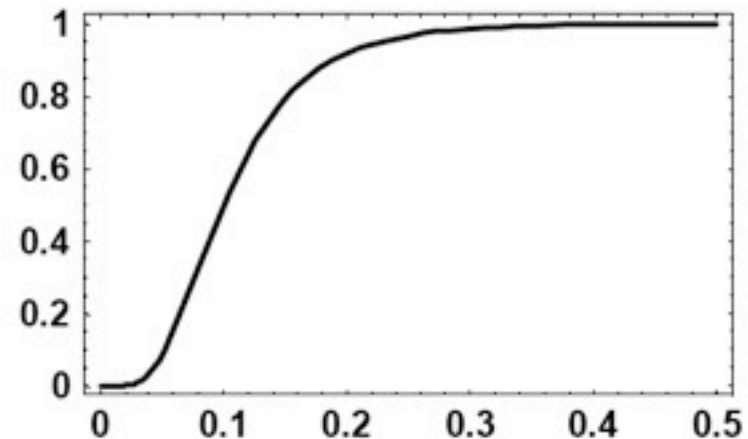
$\lambda = \text{average \# events / year} = 200$

$\mu = \ln(0.1); \sigma = 0.5$

PDF



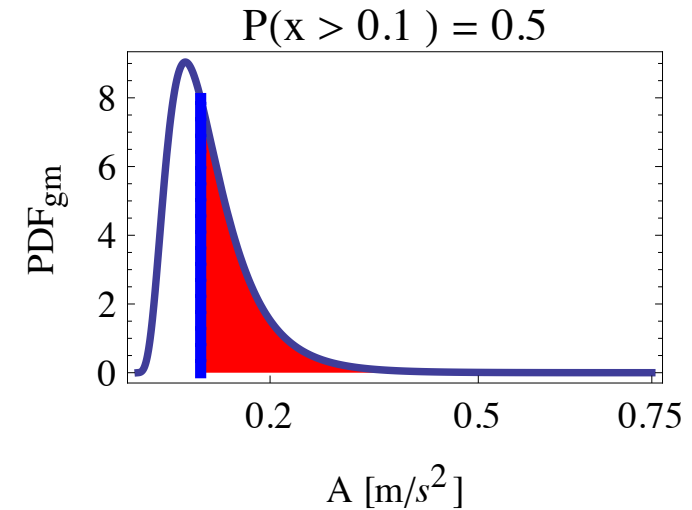
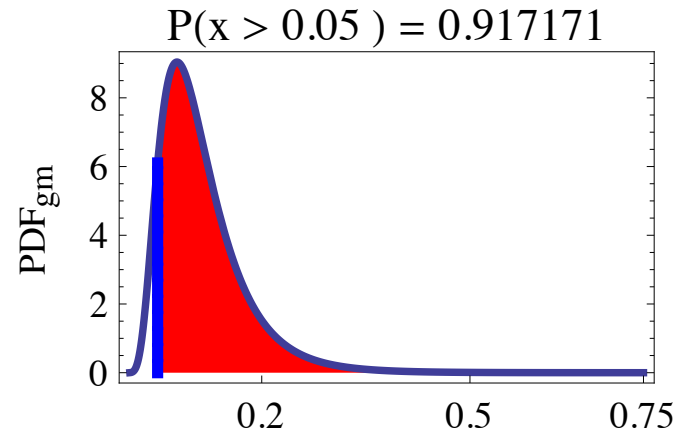
CDF



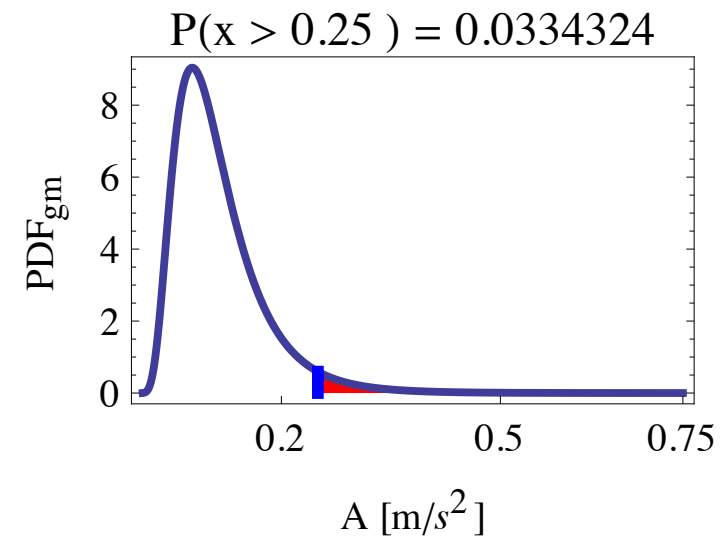
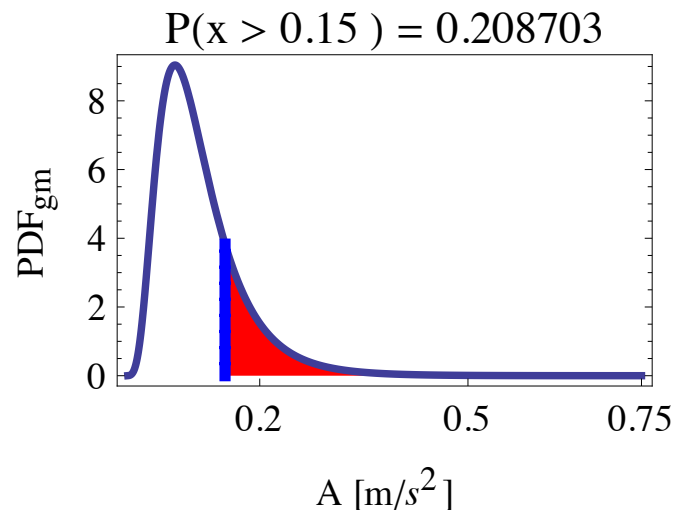
Expected daily rate of exceedance of  $x_{\text{test}}$  = Expected daily rate of occurrence of events  $(200/365 = 0.55)$   $\times P(x > x_{\text{test}} \mid \text{seismic event})$

conditional probability from GMM

# Conditional exceedance probability for different ground motion levels



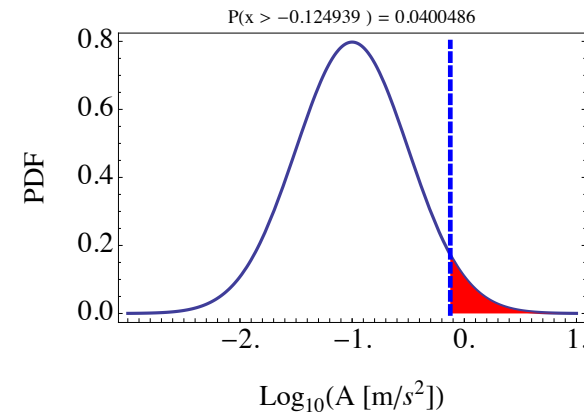
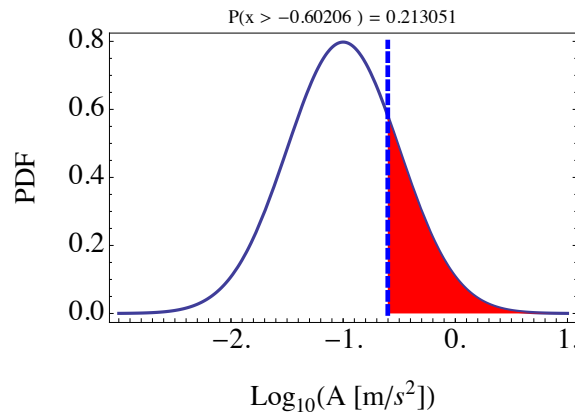
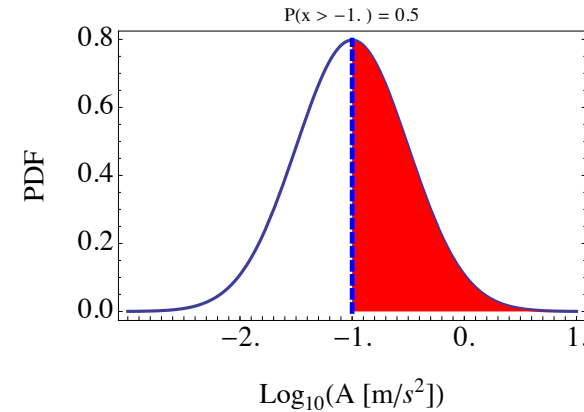
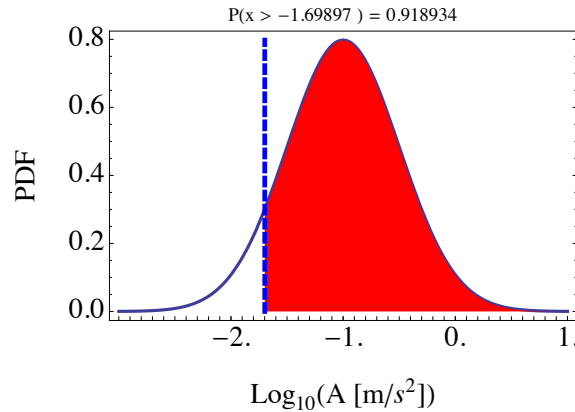
$P(x > x_{\text{test}} \mid \text{seismic event})$





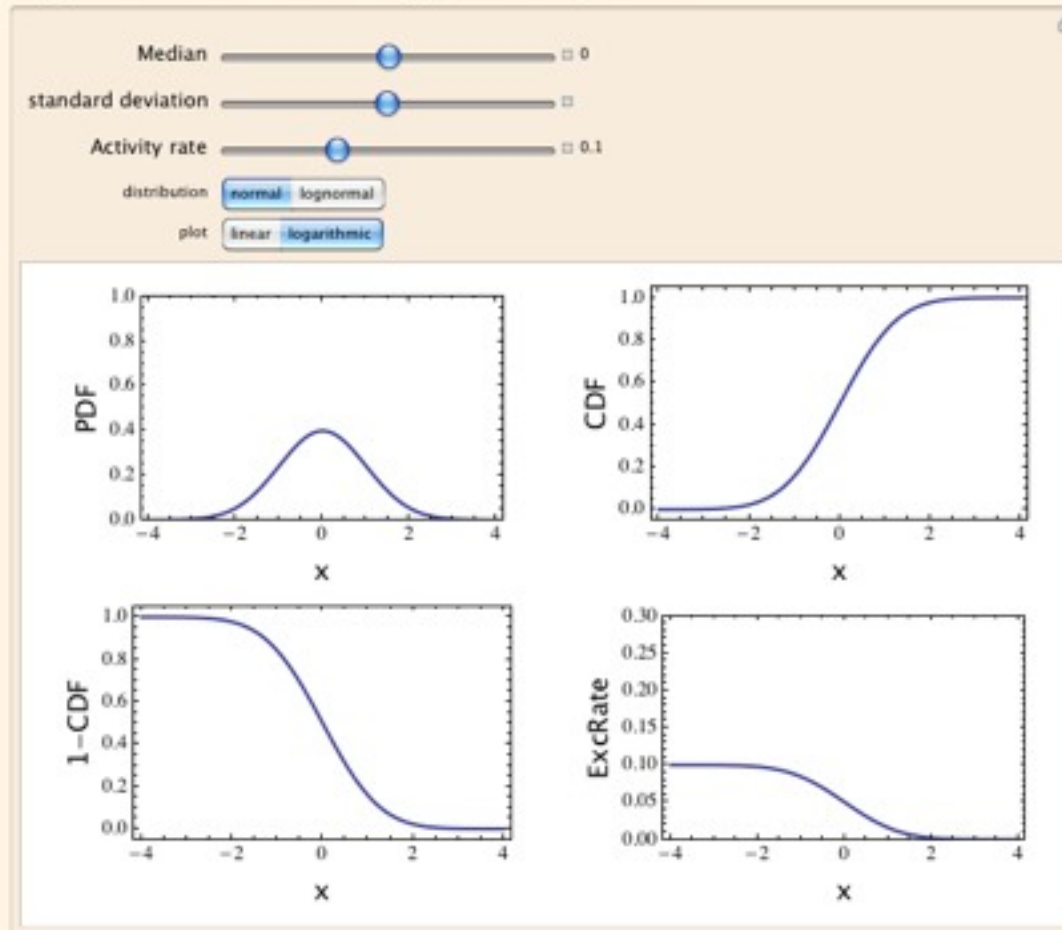
# Switching scales

$P(x > x_{\text{tes}} \mid \text{seismic event})$

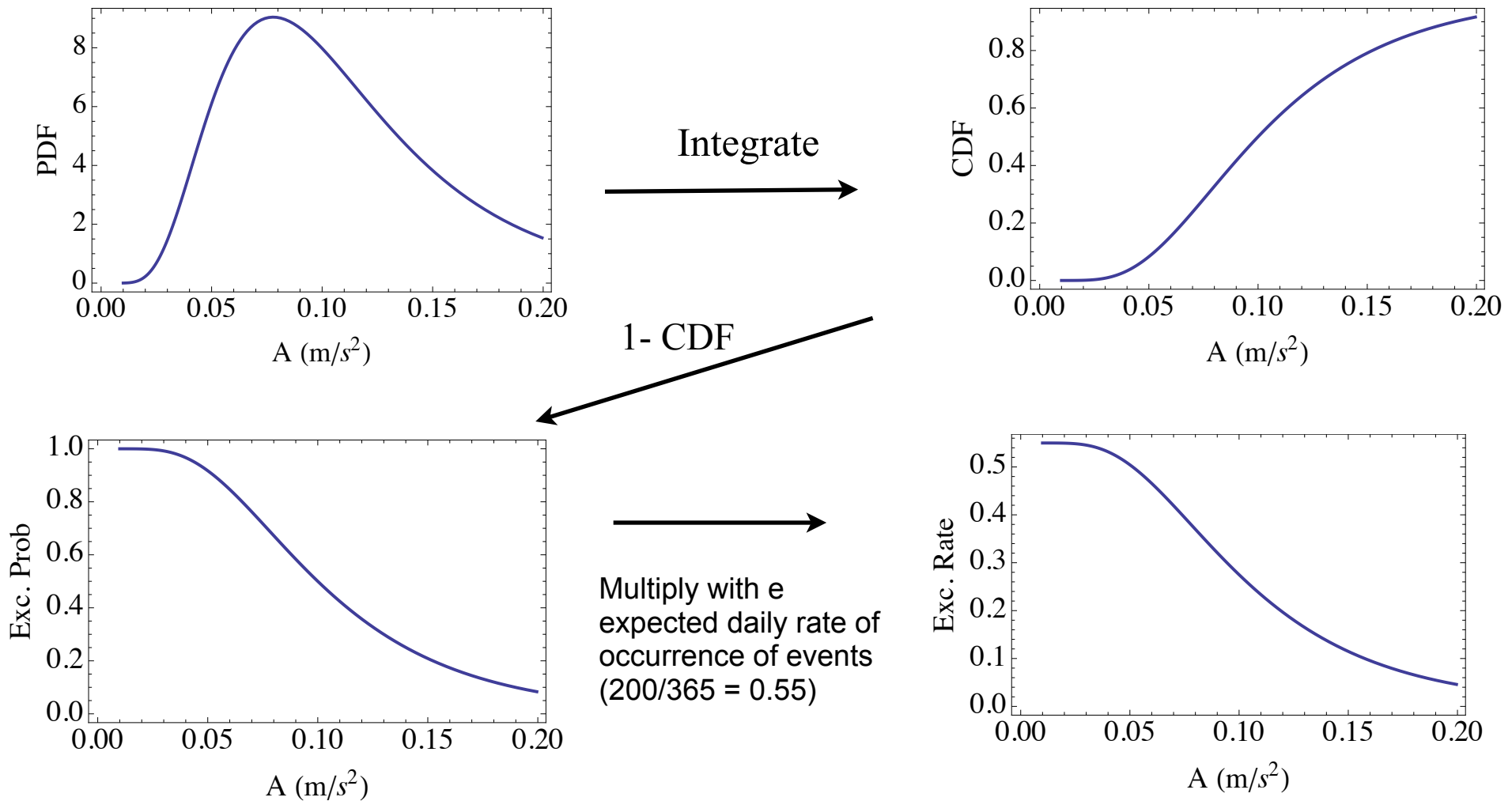


Note: x-axis now logarithmic

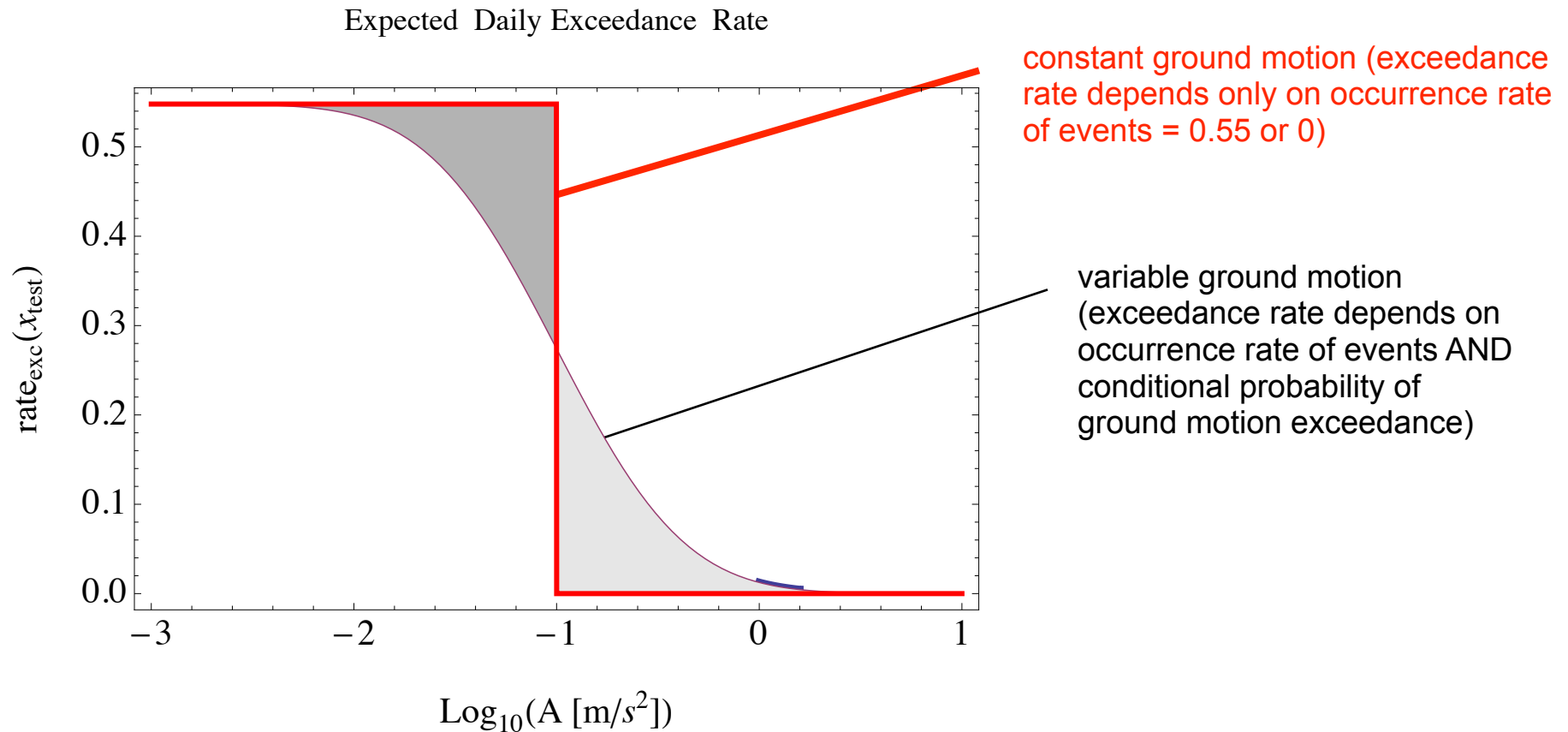
The purpose of this manipalet is to illustrate the relationships between the probability density function (PDF), the cumulative distribution function (CDF), the exceedance probability function (1-CDF) and the exceedance rate for normal and log-normally distributed random variables of varying median and sigma values.



# PDF, CDF, Exceedance Probability (Rate)



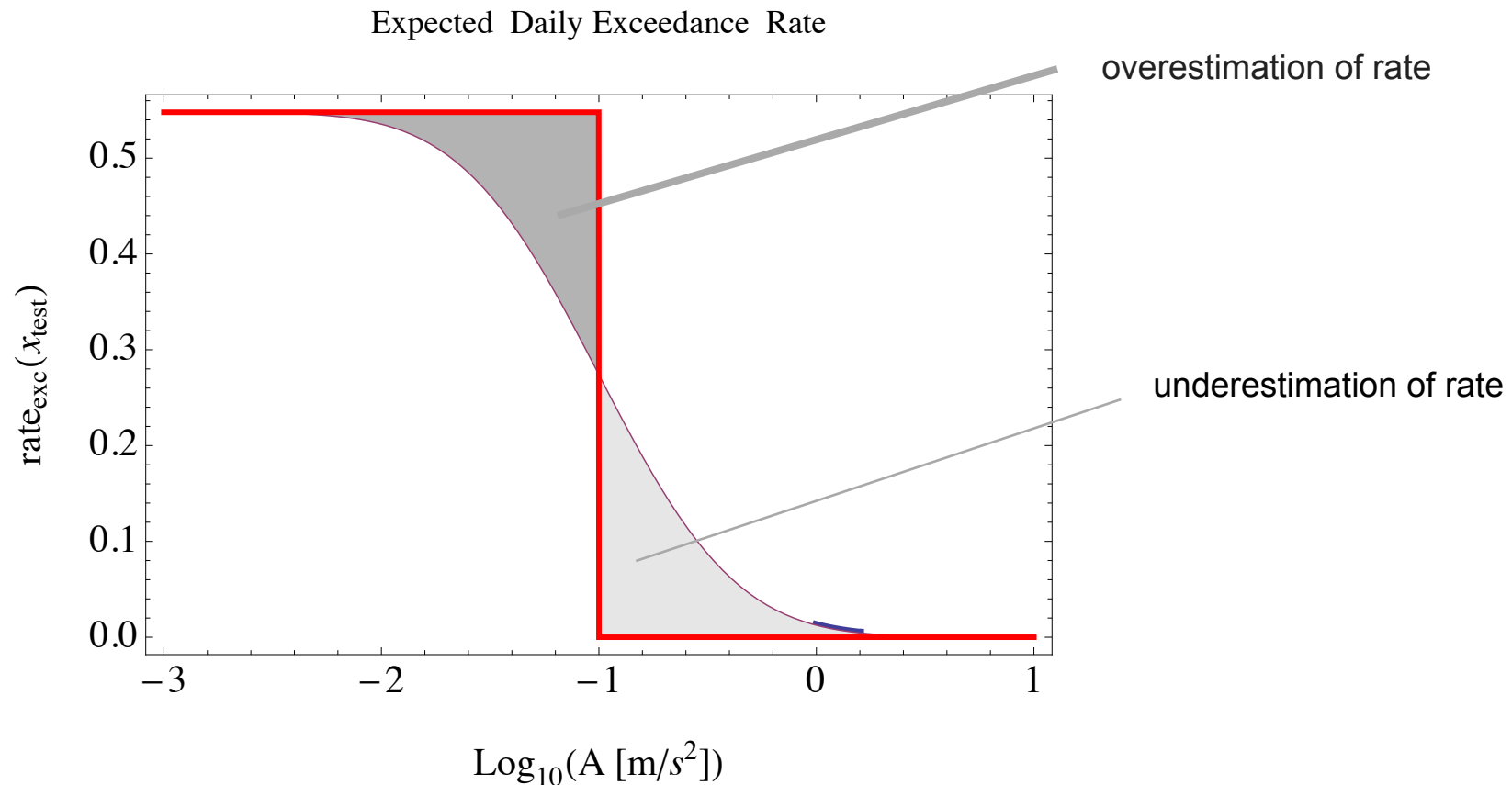
# Expected daily rate of exceedance of $x_{\text{test}}$



Expected daily rate of exceedance of  $x_{\text{test}}$  = Expected daily rate of occurrence of events (200/365 = 0.55)  $\times P(x > x_{\text{test}} \mid \text{seismic event})$

# A note on the side

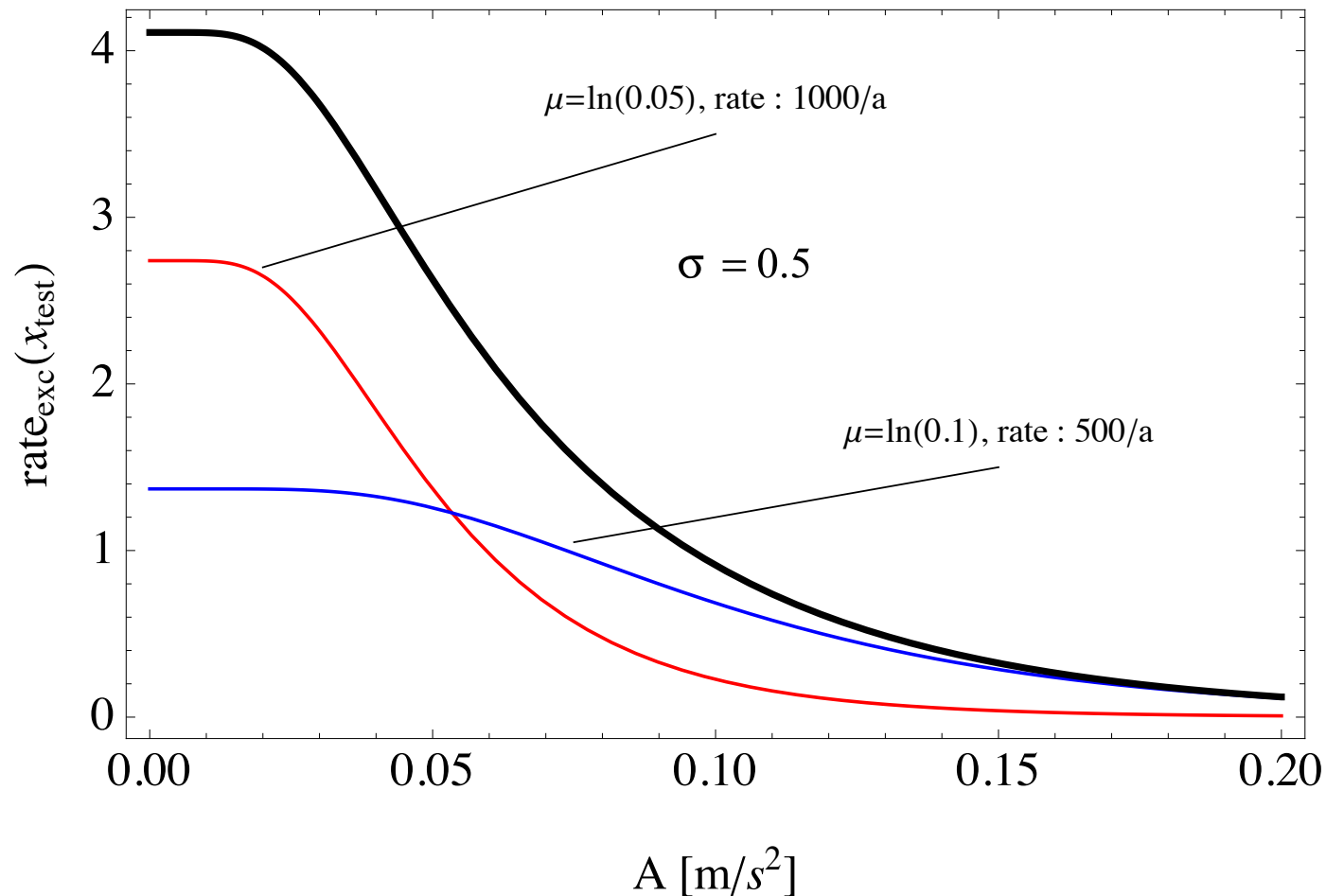
Approximating ground motion by a deterministic model will lead to overestimation of exceedance rates for low values and underestimation of high values.





# Two seismic sources with different occurrence rates

Expected Daily Exceedance Rate





# TwoSources

The purpose of this manipulet is to illustrate the superposition of two seismic sources with different activity rate and different probability density functions for their ground motion, each of which is assumed by log - normally distributed. The yellow curve represents the overall ground motion exceedance rate.

